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A Convex Surface Property

615. [March, 1966] *Proposed by Joseph Hammer, University of Sydney, Australia.*

Prove that in a three-dimensional convex surface whose volume is greater than the surface area numerically, infinitely many plane cross-sections can be found of which each area is greater than its perimeter.

Solution by Stanley Rabinowitz, Far Rockaway, New York.

Consider any line from a point on the surface to the point farthest away from it. Let this line be the z-axis, and one endpoint, the origin. Let V be the volume of the surface, S its surface area, $A(z)$ the area of any plane cross-section at height z, and $P(z)$ the perimeter of this cross-section. Then we have

$$\int_0^a A(z)dz = V$$

and

$$\int_0^a P(z)dz = S.$$

Suppose that only finitely many plane cross-sections have $A(z) > P(z)$. Since we can change the value of a function at a finite number of points without altering the value of its integral, we can redefine $A(z)$ at these points such that $A(z) \leqq P(z)$. Then for all z, $A(z) \leqq P(z)$. But upon integrating we find that $V < S$, a contradiction. Hence the theorem.