

Reprinted from the Pi Mu Epsilon Journal 4.7(1967)297.

183. Proposed by R. Penney, Ford Scientific Laboratory.

If

$$\begin{aligned}r_0(r_1r_2 + r_2r_3 + r_3r_1) &= 0, \\r_1(r_0r_2 + r_0r_3 - r_2r_3) &= 0, \\r_2(r_0r_3 + r_0r_1 - r_3r_1) &= 0, \\r_3(r_0r_1 + r_0r_2 - r_1r_2) &= 0,\end{aligned}$$

Show that at least one of the quantities r_0, r_1, r_2, r_3 vanishes.

Solution by Stanley Rabinowitz, Polytechnic Institute of Brooklyn.

If none of the r 's vanish, then the expressions in the parenthesis must vanish. These conditions may then be written as

$$\begin{aligned}(1) \quad & 1/r_1 + 1/r_2 + 1/r_3 = 0, \\(2) \quad & 1/r_2 + 1/r_3 = 1/r_0, \\(3) \quad & 1/r_3 + 1/r_1 = 1/r_0, \\(4) \quad & 1/r_1 + 1/r_2 = 1/r_0.\end{aligned}$$

Adding up (2), (3), and (4) and using (1) gives $3/r_0 = 0$, which is impossible. Hence, at least one of the r 's must vanish.