Arithmetic Progressions from Pascal's Triangle

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```
              1
            1   1
          1   2   1
        1   3   3   1
      1   4   6   4   1
    1   5  10  10   5   1
  1   6  15  20  15   6   1
1   7  21  35  35  21   7   1
```

Write down any row of Pascal's triangle. Below it, write the next row, omitting the initial "1". Divide corresponding entries of the first row by the second. The result is an arithmetic progression.

For example, rows 6 and 7 yield the following arithmetic progression:

$$\frac{1}{7},\quad \frac{6}{21},\quad \frac{15}{35},\quad \frac{20}{35},\quad \frac{15}{21},\quad \frac{6}{7},\quad \frac{1}{1}\quad .$$

The proof follows from $\binom{n}{k} = \frac{k+1}{n+1}\binom{n+1}{k+1}$.

Adjacent diagonal lines have a similar property. For example,

$$\frac{1}{1},\quad \frac{5}{4},\quad \frac{15}{10},\quad \frac{35}{20},\quad \frac{70}{35},\quad \ldots$$

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