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Central Discs and Their Centers

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Abstract. A *disc* is a circle plus its interior. If X and Y are two triangle centers, then the disc with diameter XY is known as a *central disc*. We use a computer to analyze the central discs with diameters X_iX_j for $1 \leq i < j \leq 11$, where X_n denotes the n-th triangle center cataloged in the Encyclopedia of Triangle Centers. For each disc, we determine which centers X_k $(1 \leq k \leq 100)$ must lie inside that disc and which centers must lie outside that disc. A typical result is that the center X_{37} always lies inside the disc whose diameter is X_1X_2. We also investigate acute triangles. For example, we find that in an acute triangle, X_5 always lies inside the disc whose diameter is X_2X_6.

Keywords. discs, computer-discovered mathematics, triangle centers.

Mathematics Subject Classification (2020). 51M04, 51-08.

1. Introduction

A *disc* is a circle plus its interior. If X and Y are two triangle centers, then the disc with diameter XY is known as a *central disc*.

Let X_n denote the n-th triangle center cataloged in the Encyclopedia of Triangle Centers [6].

Some central discs have been studied extensively and have been given names. For example, the disc with diameter X_2X_4 is known as the *orthocentroidal disc*.

- In [5], it was shown that the orthocentroidal disc must contain the incenter X_1.
- In [12], it was shown that the orthocentroidal disc must contain X_{13}.

- In [3], it was shown that the orthocentroidal disc must contain X_6, and X_7; and that X_{11} and X_{14} always lie outside the orthocentroidal disc.
- In [1], it was shown that X_{125} always lies outside the orthocentroidal disc.
- In [2], it was shown that X_{37}, X_{39}, X_{42}, X_{51}, and X_{381} always lie inside the orthocentroidal disc; and that X_8, X_{10}, X_{115}, and X_{125} always lie outside the orthocentroidal disc.

Some other named central discs can be found in [7] and [8] along with a list of triangle centers that must lie in these discs. A list of central discs that must contain the incenter can be found in [11].

In this paper, we will use a computer to analyze the central discs with diameters X_iX_j for $1 \le i < j \le 11$. For each disc, we determine which centers X_k ($1 \le k \le 100$) must lie inside that disc and which centers must lie outside that disc. For this study, we excluded the center X_{30} because it lies on the line at infinity.

2. Sample Theorems

Theorem 2.1. *The triangle center X_{86} must lie inside the disc with diameter X_1X_2 (Figure 1).*

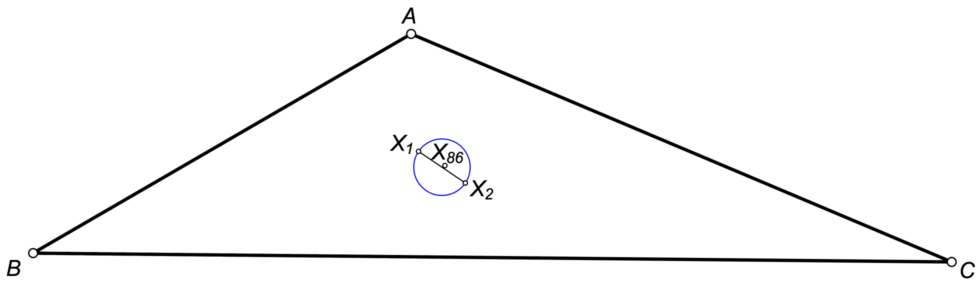

FIGURE 1. X_{86} lies inside the disc with diameter X_1X_2

Proof. From [6] we find that the barycentric coordinates for X_1 and X_2 are $X_1 = (a : b : c)$ and $X_2 = (1 : 1 : 1)$. The point X_{86} has barycentric coordinates

$$X_{86} = \big((a+b)(a+c) : (b+c)(b+a) : (c+a)(c+b)\big).$$

From the midpoint formula (equation (14) in [4]), we find that the midpoint of segment X_1X_2 has barycentric coordinates

$$M = \big(4a+b+c : 4b+c+a : 4c+a+b\big).$$

The distance, d, between X_{86} and M can be found using the distance formula (equation (9) in [4]). We find that

$$d^2 = \frac{D_1D_2}{36(a+b+c)\left(a^2+b^2+c^2+3ab+3bc+3ca\right)^2}. \tag{1}$$

where

$$D_1 = a^2b^2 + b^2c^2 + c^2a^2 - a^4 - b^4 - c^4$$

and

$$D_2 = 18abc + (a+b+c)^3 - 5(a+b+c)(ab+bc+ca).$$

The radius, r, of the disc is the distance from X_1 to M and is given by

$$r^2 = \frac{D_2}{36(a+b+c)}. \tag{2}$$

The point X_{86} will lie in the disc if and only if $d^2 \le r^2$.

This inequality can be proven using standard techniques and some ingenuity. However, we want to automatically prove the inequality by computer. This inequality is homogeneous and symmetric in a, b, c and consists of polynomials, so the inequality can be mechanically proven using Algorithm B from [10]. We have coded this algorithm as the function `proveTriangleInequalityUsingBlundon[ineq]` in Mathematica. So we issue the Mathematica command

```
proveTriangleInequalityUsingBlundon[dsq<=rsq]
```

where the variables `dsq` and `rsq` are defined with the values of d^2 and r^2 from Equations (1) and (2) above. Mathematica responds `True`, thus proving the inequality. □

Theorem 2.2. *The triangle center X_{13} must lie inside the disc with diameter X_1X_4 (Figure 2).*

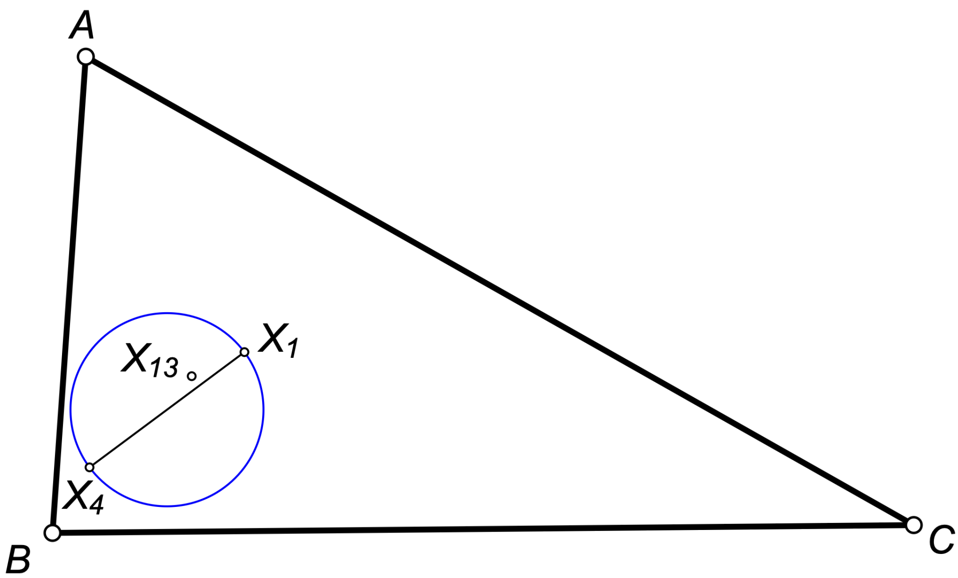

Figure 2. X_{13} lies inside the disc with diameter X_1X_4

Proof. From [6], we find that the barycentric coordinates for X_4 are

$$X_4 = \big((a^2+b^2-c^2)(a^2-b^2+c^2) ::\big)$$

where $\big(f(a,b,c) ::\big)$ means $\big(f(a,b,c) : f(b,c,a) : f(c,a,b)\big)$. Using the midpoint formula, the midpoint M of segment X_1X_4 is

$$M = \big((a+b+c)(a^4-(b^2-c^2)^2) + a(2a^2b^2+2b^2c^2+2c^2a^2-a^4-b^4-c^4 ::\big).$$

The barycentric coordinates for X_{13} are

$$X_{13} = \big(a^4 - 2(b^2-c^2)^2 + a^2(b^2+c^2+4\sqrt{3}K) ::\big)$$

where
$$K = \frac{1}{4}\sqrt{2a^2b^2 + 2b^2c^2 + 2c^2a^2 - a^4 - b^4 - c^4}$$
is the area of $\triangle ABC$.

Using the distance formula, we can find the distance, d, between X_{13} and M and the radius, r, of the disc (the distance from X_1 to M). These formulas are very complicated and will not be shown here.

The point X_{13} will lie in the disc if and only if $d^2 \leq r^2$.

Applying the function `proveTriangleInequalityUsingBlundon` to the inequality $d^2 \leq r^2$, Mathematica returns `$Failed` because algorithm B only works with symmetric homogeneous polynomial inequalities and the inequality $d^2 \leq r^2$ is not of this form due to the many square roots in the expression for d^2.

In this case, we use the slower function

```
FindInstance[dsq>rsq && triangConstraint,{a,b,c}]
```

where

```
triangConstraint = a>0 && b>0 && c>0 && a+b>c && b+c>a && c+a>b
```

restricts a, b, and c to be the sides of a triangle.

The function `FindInstance[ineq,{a,b,c}]` finds values for a, b, and c that satisfy the inequality. In this case, Mathematica responds with {} indicating that no values for a, b, and c were found. Note that the function `FindInstance` gives exact results, so a return value of {} does not mean that it didn't find any solutions; it means that there are no solutions.

Thus we can conclude that there are no values for a, b, and c such that $d^2 > r^2$. In other words, we have proven that for all triangles, $d^2 \leq r^2$. □

Theorem 2.3. *In all acute triangles, the center X_{60}, must lie inside the orthocentroidal disc (Figure 3).*

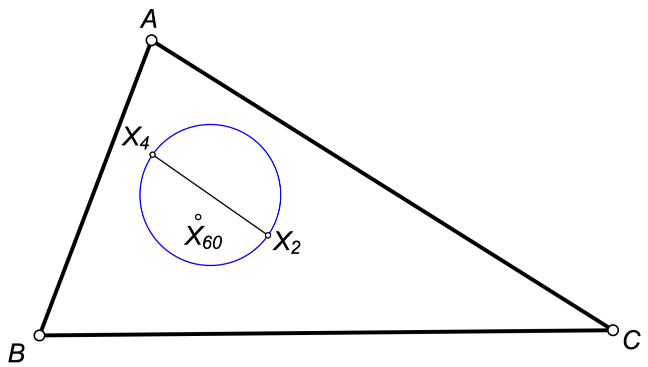

FIGURE 3. X_{60} lies inside the disc with diameter X_2X_4

Proof. From [6], we find that the coordinates for X_{60} are
$$\left(\frac{a}{\cos(B-C)} : \frac{b}{\cos(C-A)} : \frac{b}{\cos(A-B)}\right).$$
We find that M, the midpoint of X_2X_4 has barycentric coordinates
$$M = \left(a^4 - 2b^4 - 2c^4 + a^2b^2 + 4b^2c^2 + c^2a^2 ::\right).$$

Using the distance formula, we can find the distance, d, between X_{60} and M and the radius, r, of the disc (the distance from X_2 to M). Invoking the function `proveTriangleInequalityUsingBlundon` with argument $d^2 \le r^2$ produces the value `False`, showing that X_{60} does not always lie inside the orthocentroidal disc. Algorithm B can be modified to handle different shape triangles as described in [10, §15]. For the case of acute triangles, we use the code

```
proveTriangleInequalityUsingBlundon[ineq],triangleType->"acute"]
```

which checks inequalities in acute triangles.

In this case, the call

```
proveTriangleInequalityUsingBlundon[dsq<=rsq],triangleType->"acute"]
```

returns `True`, proving that in any acute triangle, X_{60} must lie in the orthocentroidal disc. □

Theorem 2.4. *In all triangles, the Mittenpunkt, X_9, must lie outside the orthocentroidal disc (Figure 4).*

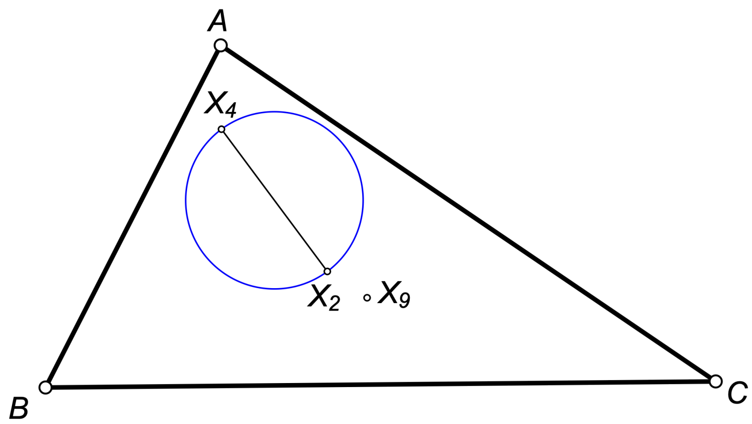

Figure 4. X_9 lies outside the disc with diameter X_2X_4

Proof. We begin the same way as in the proof of Theorem 2.1, noting that X_9 has barycentric coordinates

$$X_9 = \big(a(a-b-c) : b(-a+b-c) : c(-a-b+c)\big).$$

As before, let M denote the midpoint of X_2X_4 . Using the distance formula, we can find the distance, d, between X_9 and M and the radius, r, of the orthocentroidal disc (the distance from X_2 to M).

The point X_9 will lie outside the disc if and only if $d^2 > r^2$. So we issue the Mathematica command

```
proveTriangleInequalityUsingBlundon[dsq>rsq]
```

where the variables `dsq` and `rsq` are defined with the values of d^2 and r^2. Mathematica responds `True`, thus proving the inequality and showing that in any triangle, X_9 must lie outside the orthocentroidal disc. □

Theorem 2.5. *In all acute triangles, X_{16} must lie outside the orthocentroidal disc (Figure 5).*

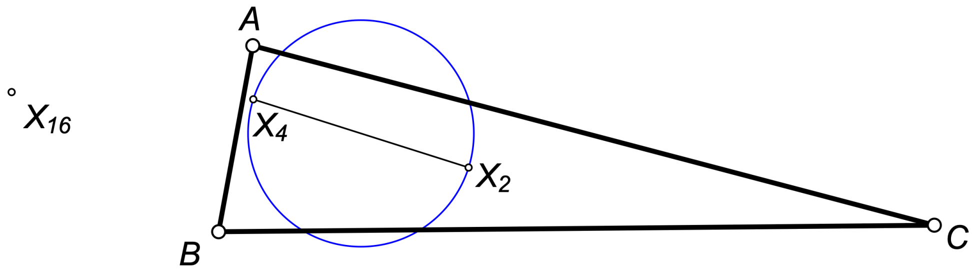

FIGURE 5. X_{16} lies outside the disc with diameter X_2X_4

Proof. From [6], we find that X_{16} has barycentric coordinates

$$X_{16} = \left(a^2(\sqrt{3}(a^2 - b^2 - c^2) + 2S) ::\right),$$

where $S = 2K$ is twice the area of $\triangle ABC$. We find that M, the midpoint of X_2X_4 has barycentric coordinates

$$M = \left(a^4 - 2b^4 - 2c^4 + a^2b^2 + 4b^2c^2 + c^2a^2 ::\right).$$

Using the distance formula, we can find the distance, d, between X_{16} and M and the radius, r, of the disc (the distance from X_2 to M). Invoking the function `proveTriangleInequalityUsingBlundon` with argument $d^2 > r^2$ produces the result `$Failed` because the inequality contains square roots and algorithm B can only handle inequalities involving polynomials.

In this case, we use the slower function

```
FindInstance[dsq<=rsq && acuteConstraint,{a,b,c}]
```

where

```
acuteConstraint = a>0 && b>0 && c>0 &&
     a^2+b^2>c^2 && b^2+c^2>a^2 && c^2+a^2>b^2
```

which restricts a, b, and c to be the sides of an acute triangle.

In this case, Mathematica responds with {} indicating that there are no values for a, b, and c that satisfy the inequality $d^2 \leq r^2$. Thus we can conclude that for all acute triangles, $d^2 > r^2$, and X_{16} must lie outside the orthocentroidal disc. □

3. RESULTS

Using the procedures given in Section 2, we examined all central discs X_iX_j with $1 \leq i < j \leq 11$. For each disc, we studied the centers X_k for $1 \leq k \leq 100$ with $k \neq i$, $k \neq j$, and $k \neq 30$. We determined whether X_k must lie in the disc for all triangles. By "in the disc" we mean "in the interior or on the boundary of the disc". We also determined whether X_k must lie outside the disc for all triangles; whether X_k must lie in the disc for all acute triangles; and whether X_k must lie outside the disc for all acute triangles. The results are given in Table 1 below.

How to read the table. The first data row of the table contains X_1X_2 in column 1. This means that the row corresponds to the disc with X_1X_2 as diameter. Column 2 in that row lists the centers that must lie in the disk for all triangles. For row 1, we see that X_{37} and X_{86} must lie in the disk. Column 3 in that row lists the additional centers that must lie in the disk if the triangles is acute. For row 1, column 3 is empty, meaning that no additional centers (other than X_{37} and X_{86}) must lie in the disk when the triangle is acute. Column 4 in that row lists the centers that must lie outside the disk in all triangles. For row 1, we see that for all triangles, $X_3, X_4, X_6, X_7, X_8, X_9, \dots$ must lie outside this disc. Column 5 in that row lists the additional centers that must lie outside the disk if the triangle is acute. For row 1, we see that $X_{13}, X_{18}, X_{24}, X_{26}, X_{47}, \dots$ must lie outside the disk in all acute triangles (in addition to the centers listed in column 4).

Table 1

Discs and their Centers				
disc with diameter	must contain these centers	and also these centers if the triangle is acute	must exclude these centers	and also these centers if the triangle is acute
X_1X_2	37, 86		3, 4, 6, 7, 8, 9, 10, 11, 14, 16, 19, 20, 21, 22, 23, 25, 27, 28, 29, 31, 32, 33, 34, 36, 40, 42, 43, 44, 46, 51, 53, 56, 57, 58, 63, 64, 65, 66, 67, 69, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 82, 84, 87, 88, 89, 90, 92, 98, 99, 100	13, 18, 24, 26, 47, 50, 52, 59, 62, 68, 70, 91, 94, 95, 96, 97
X_1X_3	2, 9, 10, 15, 21, 35, 37, 38, 45, 55, 86	48, 73, 77, 95	4, 6, 7, 16, 19, 20, 22, 23, 25, 27, 28, 31, 32, 34, 36, 40, 42, 43, 44, 46, 51, 53, 56, 57, 58, 64, 65, 74, 79, 81, 82, 84, 89, 98, 99, 100	11, 13, 14, 24, 26, 29, 33, 47, 50, 52, 62, 63, 66, 67, 68, 69, 70, 71, 72, 78, 80, 88, 90, 91, 92, 94, 96, 97
X_1X_4	6, 7, 13, 42, 51, 57, 58, 65, 79, 81, 82, 89	29, 33, 92	2, 3, 8, 9, 10, 11, 14, 20, 21, 35, 37, 38, 40, 45, 55, 63, 64, 66, 67, 69, 71, 72, 74, 75, 76, 78, 80, 84, 86, 87, 88, 90, 95, 97, 98	15, 16, 18, 19, 22, 23, 24, 25, 26, 27, 28, 34, 36, 44, 46, 47, 48, 50, 52, 53, 68, 70, 73, 77, 91, 94, 96, 99, 100

Discs and their Centers (cont.)				
disc	must contain	and if acute	must exclude	and if acute
X_1X_5	12, 17, 83, 85		2, 3, 4, 8, 9, 10, 11, 14, 20, 21, 29, 33, 39, 40, 41, 54, 60, 63, 64, 66, 67, 69, 71, 72, 74, 75, 76, 78, 80, 84, 87, 88, 90, 92, 98	6, 7, 13, 16, 18, 19, 22, 23, 24, 25, 26, 27, 28, 31, 32, 34, 36, 43, 44, 46, 47, 49, 50, 51, 52, 53, 56, 57, 58, 59, 61, 62, 65, 68, 70, 77, 79, 82, 89, 91, 93, 94, 95, 97, 99, 100
X_1X_6	42, 81		2, 3, 4, 8, 9, 10, 11, 14, 16, 19, 20, 21, 22, 23, 25, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 40, 43, 44, 45, 46, 47, 48, 51, 53, 55, 56, 57, 58, 59, 63, 64, 65, 66, 67, 69, 71, 72, 73, 74, 75, 76, 77, 78, 80, 84, 86, 87, 88, 90, 95, 97, 98, 99, 100	15, 18, 24, 26, 50, 52, 62, 68, 70, 91, 94, 96
X_1X_7		81	2, 3, 4, 8, 9, 10, 11, 14, 16, 19, 20, 21, 22, 23, 25, 27, 28, 29, 33, 34, 35, 36, 37, 38, 40, 44, 45, 46, 47, 48, 51, 55, 56, 57, 63, 64, 65, 66, 67, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 84, 86, 87, 88, 90, 92, 95, 97, 98, 99, 100	15, 18, 24, 26, 31, 32, 43, 50, 52, 53, 58, 59, 62, 68, 70, 91, 94, 96

Discs and their Centers (cont.)				
disc	must contain	and if acute	must exclude	and if acute
X_1X_8	2, 9, 10, 21, 37, 38, 45, 55, 75, 86	15, 35, 95	4, 6, 7, 11, 14, 16, 20, 31, 32, 33, 34, 36, 40, 42, 43, 44, 46, 53, 56, 57, 58, 63, 64, 65, 69, 72, 74, 78, 79, 80, 81, 82, 84, 88, 89, 90, 92, 98, 99, 100	13, 19, 22, 23, 24, 25, 26, 27, 28, 29, 47, 50, 51, 52, 62, 66, 67, 68, 70, 91, 94, 96
X_1X_9	2, 10, 21, 37, 38, 45, 55, 86	15, 35	3, 4, 6, 7, 8, 11, 14, 16, 19, 20, 31, 32, 33, 34, 36, 40, 42, 43, 44, 46, 53, 56, 57, 58, 63, 64, 65, 66, 67, 69, 71, 72, 74, 78, 79, 80, 81, 82, 84, 88, 89, 90, 92, 98, 99, 100	13, 18, 22, 23, 24, 25, 26, 27, 28, 29, 47, 50, 51, 52, 59, 62, 68, 70, 76, 91, 94, 96, 97
X_1X_{10}	2, 37, 38, 45, 86	55	3, 4, 6, 7, 8, 9, 11, 14, 16, 19, 20, 31, 32, 33, 34, 36, 40, 42, 43, 44, 46, 53, 56, 57, 58, 63, 64, 65, 66, 67, 69, 71, 72, 74, 75, 76, 78, 79, 80, 81, 82, 84, 87, 88, 89, 90, 92, 98, 99, 100	13, 18, 22, 23, 24, 25, 26, 27, 28, 29, 47, 50, 51, 52, 59, 62, 68, 70, 91, 94, 96, 97
X_1X_{11}		39, 41, 60, 61	5, 12, 14, 16, 17, 23, 36, 44, 59, 67, 74, 80, 83, 85, 88, 98, 99, 100	50, 93, 94

Discs and their Centers (cont.)				
disc	must contain	and if acute	must exclude	and if acute
X_2X_3	9, 10, 21	77, 95	1, 4, 5, 6, 7, 12, 16, 19, 20, 22, 23, 25, 27, 28, 31, 32, 34, 36, 37, 39, 40, 42, 43, 44, 46, 51, 53, 56, 57, 58, 64, 65, 74, 79, 81, 82, 83, 84, 85, 86, 89, 94, 98, 99, 100	11, 13, 14, 17, 24, 26, 29, 33, 41, 47, 50, 52, 54, 59, 60, 61, 62, 63, 66, 67, 68, 69, 70, 71, 72, 78, 80, 88, 90, 91, 92, 96, 97
X_2X_4	1, 5, 6, 7, 12, 13, 17, 37, 39, 42, 51, 57, 58, 65, 79, 81, 82, 83, 85, 86, 89	29, 33, 41, 54, 60, 61, 92	3, 8, 9, 10, 11, 14, 20, 21, 40, 63, 64, 66, 67, 69, 71, 72, 74, 75, 76, 78, 80, 84, 87, 88, 90, 98	16, 18, 19, 22, 23, 24, 25, 26, 27, 28, 34, 36, 44, 46, 47, 50, 52, 53, 68, 70, 77, 91, 94, 95, 96, 97, 99, 100
X_2X_5			3, 4, 8, 9, 10, 11, 14, 20, 21, 29, 33, 40, 63, 64, 66, 67, 69, 71, 72, 74, 75, 76, 78, 80, 84, 87, 88, 90, 92, 98	6, 7, 13, 16, 18, 19, 22, 23, 24, 25, 26, 27, 28, 31, 32, 34, 36, 43, 44, 46, 47, 50, 51, 52, 53, 56, 57, 58, 59, 62, 65, 68, 70, 77, 79, 82, 89, 91, 94, 95, 96, 97, 99, 100
X_2X_6	1, 37, 39, 42, 81, 83, 86	5, 12, 17, 41, 54, 60, 61, 85	3, 4, 8, 9, 10, 14, 16, 19, 20, 21, 22, 23, 25, 27, 28, 31, 32, 40, 43, 44, 51, 53, 58, 63, 64, 66, 67, 69, 71, 72, 74, 75, 76, 78, 84, 87, 88, 98, 99, 100	11, 18, 24, 26, 33, 34, 36, 46, 47, 50, 52, 56, 57, 59, 62, 65, 68, 70, 77, 80, 90, 91, 94, 95, 96, 97

Discs and their Centers (cont.)				
disc	must contain	and if acute	must exclude	and if acute
X_2X_7	1, 12, 37, 83, 85, 86	5, 17, 60, 61, 81	3, 4, 8, 9, 10, 11, 14, 16, 19, 20, 21, 22, 23, 25, 27, 28, 29, 36, 40, 44, 46, 51, 56, 57, 63, 64, 65, 66, 67, 69, 71, 72, 74, 75, 76, 77, 78, 79, 80, 84, 87, 88, 90, 92, 98, 99, 100	18, 24, 26, 31, 32, 33, 34, 43, 47, 50, 52, 53, 58, 59, 62, 68, 70, 91, 94, 95, 97
X_2X_8	9, 10, 21, 75	95	1, 4, 5, 6, 7, 11, 12, 14, 16, 20, 22, 23, 25, 27, 28, 31, 32, 33, 34, 36, 37, 39, 40, 42, 43, 44, 46, 51, 53, 56, 57, 58, 63, 64, 65, 69, 72, 74, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 94, 98, 99, 100	13, 17, 19, 24, 26, 29, 41, 47, 50, 52, 54, 60, 61, 62, 66, 67, 68, 70, 96
X_2X_9	10, 21		1, 3, 4, 5, 6, 7, 8, 11, 12, 14, 16, 19, 20, 22, 23, 25, 27, 28, 31, 32, 33, 34, 36, 37, 39, 40, 42, 43, 44, 46, 51, 53, 56, 57, 58, 63, 64, 65, 66, 67, 69, 71, 72, 74, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 94, 98, 99, 100	13, 17, 18, 24, 26, 29, 41, 47, 50, 52, 54, 59, 60, 61, 62, 68, 70, 76, 96, 97

Discs and their Centers (cont.)				
disc	must contain	and if acute	must exclude	and if acute
X_2X_{10}			1, 3, 4, 5, 6, 7, 8, 9, 11, 12, 14, 16, 19, 20, 22, 23, 25, 27, 28, 31, 32, 33, 34, 36, 37, 39, 40, 42, 43, 44, 46, 51, 53, 56, 57, 58, 63, 64, 65, 66, 67, 69, 71, 72, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 98, 99, 100	13, 17, 18, 24, 26, 29, 41, 47, 50, 52, 54, 59, 60, 61, 62, 68, 70, 96, 97
X_2X_{11}	38, 45, 55	15	14, 16, 23, 36, 44, 67, 74, 80, 88, 98, 99, 100	50, 59, 94
X_3X_4	1, 2, 5, 6, 7, 9, 10, 12, 13, 15, 17, 21, 35, 37, 38, 39, 41, 42, 45, 51, 55, 57, 58, 60, 61, 65, 79, 81, 82, 83, 85, 86, 89	29, 33, 48, 54, 73, 77, 92, 95	20, 40, 64, 74, 84, 98	11, 14, 16, 19, 22, 23, 24, 25, 26, 27, 28, 34, 36, 44, 46, 47, 50, 52, 53, 63, 66, 67, 68, 69, 70, 71, 72, 78, 80, 88, 90, 91, 94, 96, 97, 99, 100
X_3X_5	2, 9, 10, 21	77, 95	4, 20, 40, 64, 74, 84, 98	6, 7, 11, 13, 14, 16, 19, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 36, 43, 44, 46, 47, 50, 51, 52, 53, 56, 57, 58, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 78, 79, 80, 82, 88, 89, 90, 91, 92, 94, 96, 97, 99, 100

Discs and their Centers (cont.)				
disc	must contain	and if acute	must exclude	and if acute
X_3X_6	1, 2, 9, 10, 15, 21, 35, 37, 38, 39, 41, 42, 45, 55, 60, 61, 81, 83, 86	5, 12, 17, 48, 54, 73, 77, 85, 95	4, 16, 19, 20, 22, 23, 25, 27, 28, 31, 32, 34, 36, 40, 43, 44, 46, 51, 53, 56, 57, 58, 64, 65, 74, 84, 98, 99, 100	11, 14, 24, 26, 33, 47, 50, 52, 62, 63, 66, 67, 68, 69, 70, 71, 72, 78, 80, 88, 90, 91, 94, 96, 97
X_3X_7	1, 2, 9, 10, 12, 15, 21, 35, 37, 38, 45, 55, 60, 61, 81, 83, 85, 86	5, 17, 48, 73, 77, 95	4, 16, 19, 20, 22, 23, 25, 27, 28, 34, 36, 40, 44, 46, 51, 56, 57, 64, 65, 74, 79, 84, 98, 99, 100	11, 14, 24, 26, 29, 31, 32, 33, 43, 47, 50, 52, 53, 58, 62, 63, 66, 67, 68, 69, 70, 71, 72, 78, 80, 88, 90, 91, 92, 94, 96, 97
X_3X_8			1, 2, 4, 5, 6, 7, 9, 10, 12, 16, 19, 20, 21, 22, 23, 25, 27, 28, 31, 32, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 51, 53, 55, 56, 57, 58, 60, 64, 65, 74, 76, 79, 81, 82, 83, 84, 85, 86, 89, 94, 98, 99, 100	11, 13, 14, 15, 17, 18, 24, 26, 29, 33, 35, 47, 50, 52, 54, 61, 62, 63, 66, 67, 68, 69, 70, 72, 78, 80, 88, 90, 91, 92, 95, 96
X_3X_9			1, 2, 4, 5, 6, 7, 10, 12, 16, 19, 20, 21, 22, 23, 25, 27, 28, 31, 32, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 51, 53, 55, 56, 57, 58, 59, 60, 64, 65, 74, 79, 81, 82, 83, 84, 85, 86, 89, 94, 98, 99, 100	11, 13, 14, 15, 17, 24, 26, 29, 33, 35, 47, 50, 52, 54, 62, 63, 66, 67, 68, 69, 70, 71, 72, 78, 80, 88, 90, 91, 92, 96, 97

Discs and their Centers (cont.)				
disc	must contain	and if acute	must exclude	and if acute
X_3X_{10}	9		1, 2, 4, 5, 6, 7, 12, 16, 19, 20, 22, 23, 25, 27, 28, 31, 32, 34, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 51, 53, 56, 57, 58, 59, 64, 65, 74, 79, 81, 82, 83, 84, 85, 86, 89, 94, 98, 99, 100	11, 13, 14, 17, 24, 26, 29, 33, 41, 47, 50, 52, 54, 55, 60, 61, 62, 63, 66, 67, 68, 69, 70, 71, 72, 78, 80, 88, 90, 91, 92, 96, 97
X_3X_{11}			8, 16, 23, 36, 44, 74, 76, 98, 99, 100	14, 18, 50, 59, 67, 80, 88, 94
X_4X_5		6, 7, 13, 29, 33, 51, 65, 79, 82, 89, 92	2, 3, 8, 9, 10, 11, 14, 20, 21, 40, 50, 63, 64, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 78, 80, 84, 87, 88, 90, 98	16, 18, 19, 22, 23, 24, 25, 26, 27, 28, 34, 36, 44, 46, 47, 52, 53, 77, 91, 94, 95, 96, 97, 99, 100
X_4X_6	51, 57, 58, 65	33	1, 2, 3, 8, 9, 10, 11, 14, 20, 21, 35, 37, 38, 39, 40, 41, 42, 45, 54, 55, 60, 63, 64, 66, 67, 69, 71, 72, 74, 75, 76, 78, 80, 81, 83, 84, 86, 87, 88, 90, 95, 97, 98	5, 12, 15, 16, 17, 18, 19, 22, 23, 24, 25, 26, 27, 28, 34, 36, 44, 46, 47, 48, 50, 52, 53, 61, 68, 70, 73, 77, 85, 91, 94, 96, 99, 100
X_4X_7	51, 57, 65, 79	29, 33, 92	1, 2, 3, 8, 9, 10, 11, 12, 14, 20, 21, 35, 37, 38, 40, 45, 55, 60, 61, 63, 64, 66, 67, 69, 71, 72, 74, 75, 76, 78, 80, 81, 83, 84, 85, 86, 87, 88, 90, 95, 97, 98	5, 15, 16, 17, 18, 19, 22, 23, 24, 25, 26, 27, 28, 34, 36, 44, 46, 47, 50, 52, 53, 61, 68, 70, 73, 77, 91, 94, 96, 99, 100

Discs and their Centers (cont.)				
disc	must contain	and if acute	must exclude	and if acute
X_4X_8	1, 2, 5, 6, 7, 9, 10, 12, 13, 17, 21, 37, 38, 39, 42, 45, 51, 55, 57, 58, 65, 75, 79, 81, 82, 83, 85, 86, 89	15, 29, 33, 35, 41, 54, 60, 61, 92, 95	11,14, 20, 40, 63, 64, 66, 67, 69, 72, 74, 78, 80, 84, 88, 90, 98	16, 19, 22, 23, 24, 25, 26, 27, 28, 34, 36, 44, 46, 47, 50, 52, 53, 68, 70, 91, 94, 96, 99, 100
X_4X_9	1, 2, 5, 6, 7, 10, 12, 13, 17, 21, 37, 38, 39, 42, 45, 51, 55, 57, 58, 65, 79, 81, 82, 83, 85, 86, 89	15, 29, 33, 35, 41, 54, 60, 61, 92	3, 8, 11, 14, 20, 40, 63, 64, 66, 67, 69, 71, 72, 74, 78, 80, 84, 90, 98	16, 18, 19, 22, 23, 24, 25, 26, 27, 28, 34, 36, 44, 46, 47, 50, 52, 53, 68, 70, 76, 88, 91, 94, 96, 97, 99, 100
X_4X_{10}	1, 2, 5, 6, 7, 12, 13, 17, 37, 38, 39, 42, 45, 51, 57, 58, 65, 79, 81, 82, 83, 85, 86, 89	29, 33, 41, 54, 55, 60, 61, 92	3, 8, 9, 11, 14, 20, 40, 63, 64, 66, 67, 69, 71, 72, 74, 75, 76, 78, 80, 84, 87, 88, 90, 98	16, 18, 19, 22, 23, 24, 25, 26, 27, 28, 34, 36, 44, 46, 47, 50, 52, 53, 68, 70, 91, 94, 96, 97, 99, 100
X_4X_{11}	32, 56		14, 67, 74, 80, 98	16, 23, 36, 44, 88, 94, 99, 100
X_5X_6			2, 3, 4, 8, 9, 10, 11, 14, 20, 21, 33, 40, 63, 64, 66, 67, 69, 71, 72, 74, 75, 76, 78, 80, 84, 87, 88, 90, 98	16, 18, 19, 22, 23, 24, 25, 26, 27, 28, 31, 32, 34, 36, 43, 44, 46, 47, 50, 51, 52, 53, 56, 57, 58, 59, 62, 65, 68, 70, 77, 91, 94, 95, 96, 97, 99, 100
X_5X_7			2, 3, 4, 8, 9, 10, 11, 14, 20, 21, 29, 33, 40, 63, 64, 66, 67, 69, 71, 72, 74, 75, 76, 78, 80, 84, 87, 88, 90, 92, 98	16, 18, 19, 22, 23, 24, 25, 26, 27, 28, 31, 32, 34, 36, 43, 44, 46, 47, 50, 51, 52, 56, 57, 58, 59, 62, 65, 68, 70, 77, 79, 91, 94, 95, 96, 97, 99, 100

Discs and their Centers (cont.)				
disc	must contain	and if acute	must exclude	and if acute
X_5X_8	2, 9, 10, 21, 75	95	4, 11, 14, 20, 33, 40, 63, 64, 66, 69, 72, 74, 78, 80, 84, 88, 90, 92, 98	6, 7, 13, 16, 19, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 34, 36, 43, 44, 46, 47, 50, 51, 52, 53, 56, 57, 58, 62, 65, 67, 68, 70, 79, 82, 89, 91, 94, 96, 99, 100
X_5X_9	2, 10, 21		3, 4, 8, 11, 14, 20, 33, 40, 63, 64, 66, 67, 69, 71, 72, 74, 78, 80, 84, 88, 90, 92, 98	6, 7, 13, 16, 18, 19, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 34, 36, 43, 44, 46, 47, 50, 51, 52, 53, 56, 57, 58, 62, 65, 68, 70, 76, 79, 82, 89, 91, 94, 96, 97, 99, 100
X_5X_{10}	2		3, 4, 8, 9, 11, 14, 20, 33, 40, 63, 64, 66, 67, 69, 71, 72, 74, 75, 76, 78, 80, 84, 87, 88, 90, 92, 98	6, 7, 13, 16, 18, 19, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 34, 36, 43, 44, 46, 47, 50, 51, 52, 53, 56, 57, 58, 62, 65, 68, 70, 79, 82, 89, 91, 94, 96, 97, 99, 100
X_5X_{11}	1, 12, 17, 39, 83, 85	41, 54, 60	14, 67, 74, 80, 88, 98	16, 23, 36, 44, 50, 59, 93, 94, 99, 100

Discs and their Centers (cont.)				
disc	must contain	and if acute	must exclude	and if acute
X_6X_7	82, 89		1, 2, 3, 4, 8, 9, 10, 11, 14, 16, 19, 20, 21, 22, 23, 25, 27, 28, 33, 34, 35, 36, 37, 38, 40, 44, 45, 46, 47, 48, 51, 55, 59, 60, 63, 64, 66, 67, 69, 71, 72, 73, 74, 75, 76, 77, 78, 80, 83, 84, 86, 87, 88, 90, 95, 97, 98, 99, 100	5, 12, 13, 15, 17, 18, 24, 26, 31, 32, 43, 50, 52, 53, 56, 57, 58, 61, 62, 65, 68, 70, 81, 85, 91, 94, 96
X_6X_8	1, 2, 9, 10, 21, 37, 38, 39, 42, 45, 55, 75, 81, 83, 86	5, 12, 15, 17, 35, 41, 54, 60, 61, 85, 95	4, 14, 16, 20, 31, 32, 40, 43, 44, 53, 58, 63, 64, 69, 72, 74, 78, 84, 88, 98, 99, 100	11, 19, 22, 23, 24, 25, 26, 27, 28, 33, 34, 36, 46, 47, 50, 51, 52, 56, 57, 62, 65, 66, 67, 68, 70, 80, 90, 91, 94, 96
X_6X_9	1, 2, 10, 21, 37, 38, 39, 42, 45, 55, 81, 83, 86	5, 12, 15, 17, 35, 41, 54, 60, 61, 85	3, 4, 8, 14, 16, 19, 20, 31, 32, 40, 43, 44, 53, 58, 63, 64, 66, 67, 69, 71, 72, 74, 78, 84, 88, 98, 99, 100	11, 18, 22, 23, 24, 25, 26, 27, 28, 33, 34, 36, 46, 47, 50, 51, 52, 56, 57, 62, 65, 68, 70, 76, 80, 90, 91, 94, 96, 97
X_6X_{10}	1, 2, 37, 38, 39, 42, 45, 81, 83, 86	5, 12, 17, 41, 54, 55, 60, 61, 85	3, 4, 8, 9, 14, 16, 19, 20, 31, 32, 40, 43, 44, 53, 58, 63, 64, 66, 67, 69, 71, 72, 74, 75, 76, 78, 84, 87, 88, 98, 99, 100	11, 18, 22, 23, 24, 25, 26, 27, 28, 33, 34, 36, 46, 47, 50, 51, 52, 56, 57, 62, 65, 68, 70, 80, 90, 91, 94, 96, 97
X_6X_{11}			7, 13, 14, 16, 23, 36, 44, 59, 67, 74, 80, 88, 98, 99, 100	50, 82, 89, 94

Discs and their Centers (cont.)				
disc	must contain	and if acute	must exclude	and if acute
X_7X_8	1, 2, 9, 10, 12, 21, 37, 38, 45, 55, 75, 83, 85, 86	5, 15, 17, 35, 60, 61, 81, 95	4, 11, 14, 16, 20, 36, 40, 44, 46, 56, 57, 63, 64, 65, 69, 72, 74, 78, 79, 80, 84, 88, 90, 92, 98, 99, 100	19, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 43, 47, 50, 51, 52, 53, 58, 62, 66, 67, 68, 70, 91, 94, 96
X_7X_9	1, 2, 10, 12, 21, 37, 38, 45, 55, 83, 85, 86	5, 15, 17, 35, 60, 61, 81	3, 4, 8, 11, 14, 16, 19, 20, 36, 40, 44, 46, 56, 57, 63, 64, 65, 66, 67, 69, 71, 72, 74, 78, 79, 80, 84, 88, 90, 92, 98, 99, 100	18, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 43, 47, 50, 51, 52, 53, 58, 62, 68, 70, 76, 91, 94, 96, 97
X_7X_{10}	1, 2, 12, 37, 38, 45, 83, 85, 86	5, 17, 55, 60, 61, 81	3, 4, 8, 9, 11, 14, 16, 19, 20, 36, 40, 44, 46, 56, 57, 63, 64, 65, 66, 67, 69, 71, 72, 74, 75, 76, 78, 79, 80, 84, 87, 88, 90, 92, 98, 99, 100	18, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 43, 47, 50, 51, 52, 53, 58, 62, 68, 70, 91, 94, 96, 97
X_7X_{11}		6, 82, 89	13, 14, 16, 23, 36, 44, 59, 67, 74, 80, 88, 98, 99, 100	50, 94
X_8X_9			1, 2, 4, 5, 6, 7, 10, 11, 12, 14, 16, 19, 20, 21, 22, 23, 25, 27, 28, 31, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 51, 53, 55, 56, 57, 58, 60, 63, 64, 65, 69, 72, 74, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 94, 98, 99, 100	13, 15, 17, 24, 26, 29, 35, 47, 50, 52, 54, 61, 62, 66, 67, 68, 70, 96

Discs and their Centers (cont.)				
disc	must contain	and if acute	must exclude	and if acute
X_8X_{10}	9, 75		1, 2, 4, 5, 6, 7, 11, 12, 14, 16, 19, 20, 22, 23, 25, 27, 28, 31, 32, 33, 34, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 51, 53, 56, 57, 58, 63, 64, 65, 69, 72, 74, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 94, 98, 99, 100	13, 17, 24, 26, 29, 41, 47, 50, 52, 54, 55, 60, 61, 62, 66, 67, 68, 70, 96
X_8X_{11}		3	14, 16, 36, 44, 74, 76, 80, 88, 98, 99, 100	18, 23, 50, 67, 94
X_9X_{10}			1, 2, 3, 4, 5, 6, 7, 8, 11, 12, 14, 16, 19, 20, 22, 23, 25, 27, 28, 29, 31, 32, 33, 34, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 51, 53, 56, 57, 58, 63, 64, 65, 66, 67, 69, 71, 72, 74, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 94, 98, 99, 100	13, 17, 18, 24, 26, 41, 47, 50, 52, 54, 55, 59, 60, 61, 62, 68, 70, 76, 96, 97
X_9X_{11}		48, 95	14, 16, 36, 44, 67, 74, 75, 80, 87, 88, 98, 99, 100	23, 50, 59, 94
$X_{10}X_{11}$	21	35, 73	14, 16, 36, 44, 67, 74, 80, 88, 98, 99, 100	23, 50, 59, 94

The data is represented differently in the following table. In Table 2, the first column lists a triangle center. The second column lists all central disks that always contain this center. The central disk is specified by giving its diameter. For example, from row 1 of the table, we see that center X_1 must always lie in the disc with diameter X_2X_4.

Table 2

Centers that must be in Discs	
center	diameter of disc
X_1	X_2X_4, X_2X_6, X_2X_7, X_3X_4, X_3X_6, X_3X_7, X_4X_8, X_4X_9, X_4X_{10}, X_5X_{11}, X_6X_8, X_6X_9, X_6X_{10}, X_7X_8, X_7X_9, X_7X_{10}
X_2	X_1X_3, X_1X_8, X_1X_9, X_1X_{10}, X_3X_4, X_3X_5, X_3X_6, X_3X_7, X_4X_8, X_4X_9, X_4X_{10}, X_5X_8, X_5X_9, X_5X_{10}, X_6X_8, X_6X_9, X_6X_{10}, X_7X_8, X_7X_9, X_7X_{10}
X_5	X_2X_4, X_3X_4, X_4X_8, X_4X_9, X_4X_{10}
X_6	X_1X_4, X_2X_4, X_3X_4, X_4X_8, X_4X_9, X_4X_{10}
X_7	X_1X_4, X_2X_4, X_3X_4, X_4X_8, X_4X_9, X_4X_{10}
X_9	X_1X_3, X_1X_8, X_2X_3, X_2X_8, X_3X_4, X_3X_5, X_3X_6, X_3X_7, X_3X_{10}, X_4X_8, X_5X_8, X_6X_8, X_7X_8, X_8X_{10}
X_{10}	X_1X_3, X_1X_8, X_1X_9, X_2X_3, X_2X_8, X_2X_9, X_3X_4, X_3X_5, X_3X_6, X_3X_7, X_4X_8, X_4X_9, X_5X_8, X_5X_9, X_6X_8, X_6X_9, X_7X_8, X_7X_9
X_{12}	X_1X_5, X_2X_4, X_2X_7, X_3X_4, X_3X_7, X_4X_8, X_4X_9, X_4X_{10}, X_5X_{11}, X_7X_8, X_7X_9, X_7X_{10}
X_{21}	X_1X_3, X_1X_8, X_1X_9, X_2X_3, X_2X_8, X_2X_9, X_3X_4, X_3X_5, X_3X_6, X_3X_7, X_4X_8, X_4X_9, X_5X_8, X_5X_9, X_6X_8, X_6X_9, X_7X_8, X_7X_9, $X_{10}X_{11}$
X_{32}	X_4X_{11}
X_{35}	X_1X_3, X_3X_4, X_3X_6, X_3X_7
X_{37}	X_1X_2, X_1X_3, X_1X_8, X_1X_9, X_1X_{10}, X_2X_4, X_2X_6, X_2X_7, X_3X_4, X_3X_6, X_3X_7, X_4X_8, X_4X_9, X_4X_{10}, X_6X_8, X_6X_9, X_6X_{10}, X_7X_8, X_7X_9, X_7X_{10}
X_{38}	X_1X_3, X_1X_8, X_1X_9, X_1X_{10}, X_2X_{11}, X_3X_4, X_3X_6, X_3X_7, X_4X_8, X_4X_9, X_4X_{10}, X_6X_8, X_6X_9, X_6X_{10}, X_7X_8, X_7X_9, X_7X_{10}
X_{39}	X_2X_4, X_2X_6, X_3X_4, X_3X_6, X_4X_8, X_4X_9, X_4X_{10}, X_5X_{11}, X_6X_8, X_6X_9, X_6X_{10}
X_{41}	X_3X_4, X_3X_6
X_{42}	X_1X_4, X_1X_6, X_2X_4, X_2X_6, X_3X_4, X_3X_6, X_4X_8, X_4X_9, X_4X_{10}, X_6X_8, X_6X_9, X_6X_{10}
X_{45}	X_1X_3, X_1X_8, X_1X_9, X_1X_{10}, X_2X_{11}, X_3X_4, X_3X_6, X_3X_7, X_4X_8, X_4X_9, X_4X_{10}, X_6X_8, X_6X_9, X_6X_{10}, X_7X_8, X_7X_9, X_7X_{10}
X_{51}	X_1X_4, X_2X_4, X_3X_4, X_4X_6, X_4X_7, X_4X_8, X_4X_9, X_4X_{10}
X_{55}	X_1X_3, X_1X_8, X_1X_9, X_2X_{11}, X_3X_4, X_3X_6, X_3X_7, X_4X_8, X_4X_9, X_6X_8, X_6X_9, X_7X_8, X_7X_9
X_{56}	X_4X_{11}
X_{57}	X_1X_4, X_2X_4, X_3X_4, X_4X_6, X_4X_7, X_4X_8, X_4X_9, X_4X_{10}

Centers that must be in Discs (cont.)	
center	diameter of disc
X_{58}	X_1X_4, X_2X_4, X_3X_4, X_4X_6, X_4X_8, X_4X_9, X_4X_{10}
X_{60}	X_3X_4, X_3X_6, X_3X_7
X_{65}	X_1X_4, X_2X_4, X_3X_4, X_4X_6, X_4X_7, X_4X_8, X_4X_9, X_4X_{10}
X_{75}	X_1X_8, X_2X_8, X_4X_8, X_5X_8, X_6X_8, X_7X_8, X_8X_{10}
X_{79}	X_1X_4, X_2X_4, X_3X_4, X_4X_7, X_4X_8, X_4X_9, X_4X_{10}
X_{81}	X_1X_4, X_1X_6, X_2X_4, X_2X_6, X_3X_4, X_3X_6, X_3X_7, X_4X_8, X_4X_9, X_4X_{10}, X_6X_8, X_6X_9, X_6X_{10}
X_{82}	X_1X_4, X_2X_4, X_3X_4, X_4X_8, X_4X_9, X_4X_{10}, X_6X_7
X_{83}	X_1X_5, X_2X_4, X_2X_6, X_2X_7, X_3X_4, X_3X_6, X_3X_7, X_4X_8, X_4X_9, X_4X_{10}, X_5X_{11}, X_6X_8, X_6X_9, X_6X_{10}, X_7X_8, X_7X_9, X_7X_{10}
X_{85}	X_1X_5, X_2X_4, X_2X_7, X_3X_4, X_3X_7, X_4X_8, X_4X_9, X_4X_{10}, X_5X_{11}, X_7X_8, X_7X_9, X_7X_{10}
X_{86}	X_1X_2, X_1X_3, X_1X_8, X_1X_9, X_1X_{10}, X_2X_4, X_2X_6, X_2X_7, X_3X_4, X_3X_6, X_3X_7, X_4X_8, X_4X_9, X_4X_{10}, X_6X_8, X_6X_9, X_6X_{10}, X_7X_8, X_7X_9, X_7X_{10}
X_{89}	X_1X_4, X_2X_4, X_3X_4, X_4X_8, X_4X_9, X_4X_{10}, X_6X_7

4. Areas for Future Research

Look for patterns

Look for patterns within the data covered by this study.

Use other triangles

Extend this study to use other triangles, such as obtuse triangles, right triangles, and heptagonal triangles.

Look for discs that contain other discs

Let X_a, X_b, X_c, and X_d be non-collinear triangle centers. Find examples of a, b, c, and d such that in all triangles, the disc with diameter X_cX_d always lies inside the disc with diameter X_aX_b.

Find sharper inequalities

While drawing Figure 1 for Theorem 2.1, the author found that not only does X_{86} lie in the disc with diameter X_1X_2, but X_{86} always seems to be "close" to segment X_1X_2. Give a more precise statement of this fact and find corresponding numerical inequalities. For example, the following result was found by computer.

Theorem 4.1. *Let d be the distance from X_{86} to line segment X_1X_2 (Figure 1). Then $d \leq D/10$ where D is the length of X_1X_2.*

Find similar results for other points and central discs.

Look at conjugates

Lukarevsky [7] observed that the incenter must lie in the disc with diameter X_2X_6 and that centers X_2 and X_6 are isogonal conjugates. He conjectured that if P is any point inside $\triangle ABC$ and Q is the isogonal conjugate of P, then the disc with diameter PQ must contain the incenter. A proof can be found in [11].

Lukarevsky [9] observed that the centroid must lie in the disc with diameter X_7X_8 and that centers X_7 and X_8 are isotomic conjugates. He conjectured that if P is any point inside $\triangle ABC$ and Q is the isotomic conjugate of P, then the disc with diameter PQ must contain the centroid. This conjecture was disproven in Figure 2 of [9].

Look at other conjugates P and Q (such as Ceva conjugate, cross conjugate, cevapoint, isoconjugate, crosspoint, cyclocevian conjugate, inverse in circumcircle, etc.) and determine if the fixed point of the conjugation must always lie in the disc with diameter PQ.

Replace the disc by another figure

Let X and Y be triangle centers. Instead of looking at a disc with diameter XY, look at other figures, such as a square with diagonal XY.

Replace the disc by a triangle

Let X, Y, and Z be triangle centers. Determine which triangle centers must lie inside $\triangle XYZ$.

See if center can fill disc

In [5], it is shown that not only does X_1 lie in the orthocentroidal disc, but X_1 can be located at almost any point in the interior of the orthocentroidal disc. This is not true for other discs. For example, X_{86} must lie in the disc with diameter X_1X_2, but it can not get very far away from that diameter. Find other discs in which a center must lie and where that center can lie at almost any place within the interior of the disk and/or determine the region within the disc wherein the center must lie.

Find inequalities for empty discs

The disc with diameter X_2X_5 need not contain any center from X_1 through X_{100}. So in some sense, this disc must be very small compared to the triangle. Give a more precise statement of this fact and find corresponding numerical inequalities. For example, the following result was found by computer.

Theorem 4.2. *Let r be the radius of the disc with diameter X_2X_5. Let R be the radius of the circumcircle of $\triangle ABC$. Then $r \leq R/4$.*

Find similar results for other central discs.

Look for non-computer proofs

For any of the results in this paper, see if a non-computer proof can be found.

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