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Comment on Problem 637

637. [November, 1966, and May, 1967] *Proposed by Stanley Rabinowitz, Far Rockaway, New York.*

Prove that a triangle is isosceles if and only if it has two equal symmedians.

Comment by the proposer.

The published solution only proves half of the theorem. The fact that a and b can be interchanged without affecting the equality only proves that if $a=b$, then $k_a=k_b$.

Conversely, if $k_a=k_b$, then $k_a{}^2=k_b{}^2$ which implies that $(b^2-a^2)[4a^2b^2c^2+2c^4(b^2+a^2)+2c^6+a^2b^2(b^2+a^2)]=0$, which for positive a, b, c implies that $a=b$.

To see the need for this, suppose

$$k_a = a(a+b-2c)$$
$$k_b = b(a+b-2c),$$

then a and b can be interchanged without affecting the equality but k_a can equal k_b even if $a\neq b$, i.e., when $a+b=2c$; because $k_a-k_b=(a-b)(a+b-2c)$.