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Cyclologic Triangles Formed by Three Isodynamic Points

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Abstract. Let D be any point inside $\triangle ABC$. Let E, F, and G be the $X(15)$-points of triangles BCD, CAD, and ABD, respectively. We give a geometric proof that triangles ABC and EFG are cyclologic, i.e. the circles $\odot AFG$, $\odot BGE$, and $\odot CEF$ pass through a common point.

Keywords. triangle centers, computer-discovered mathematics, Euclidean geometry, Isodynamic points, Rabinowitz-Suppa Triangles.

Mathematics Subject Classification (2020). 51M04, 51-08.

1. Introduction

In a recent paper [4], the following result was stated.

Theorem 1.1. *Let D be any point inside $\triangle ABC$. Let E, F, and G be the $X(15)$-points of triangles BCD, CAD, and ABD, respectively (Figure 1). Then triangles ABC and EFG are cyclologic, i.e. the circles $\odot AFG$, $\odot BGE$, and $\odot CEF$ pass through a common point.*

The result was found experimentally, but no formal proof was given. In this paper, we give a geometric proof of this result.

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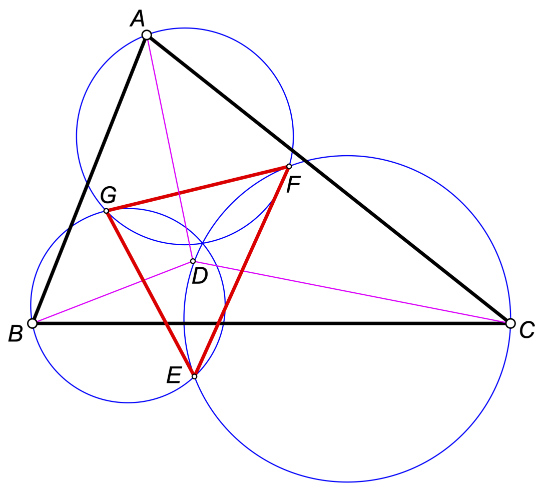


FIGURE 1. Cyclologic triangles ABC and EFG

2. NOTATION

The symbol X_n denotes the n-th named point cataloged in the Encyclopedia of Triangle Centers [1].

In particular, X_{15} is the 1st isodynamic point of a triangle [3].

Let $T = ABC$ and $T' = A'B'C'$ be two non-degenerate triangles. It is known [4] that if circles $\odot AB'C'$, $\odot BC'A'$, and $\odot CA'B'$ concur in a point P then the circles $\odot A'BC$, $\odot B'CA$, and $\odot C'AB$ concur in a point P', and reciprocally. In this case, triangles T and T' are said to be *cyclologic triangles*. The points P and P' are called the *cyclologic centers* of the triangles [2].

3. PRELIMINARY RESULTS

The following result is well known [3, Property 1.1.3].

Lemma 3.1. *The 1st isodynamic point of $\triangle ABC$ lies inside the circumcircle of $\triangle ABC$.*

The following result is well known [3, Property 1.4].

Lemma 3.2. *Let S be the 1st isodynamic point of $\triangle ABC$. Then A is the 2nd isodynamic point of $\triangle BCS$.*

The following result is well known [3, Property 4.3.4].

Lemma 3.3. *Let S' be the 2nd isodynamic point of $\triangle ABC$. Let A', B', and C' be the inverses [5] of points A, B, and C under an inversion with S' as center and any radius. Then $A'B'C'$ is an equilateral triangle (Figure 2).*

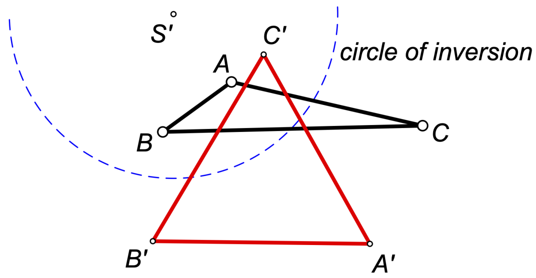


Figure 2. Inverse of $\triangle ABC$ about S'

Lemma 3.4. *Let E be the 1st isodynamic point of $\triangle BCD$. Let B', C', and E' be the inverses of points B, C, and E under an inversion with D as center and any radius. Then $B'C'E'$ is an equilateral triangle (Figure 3).*

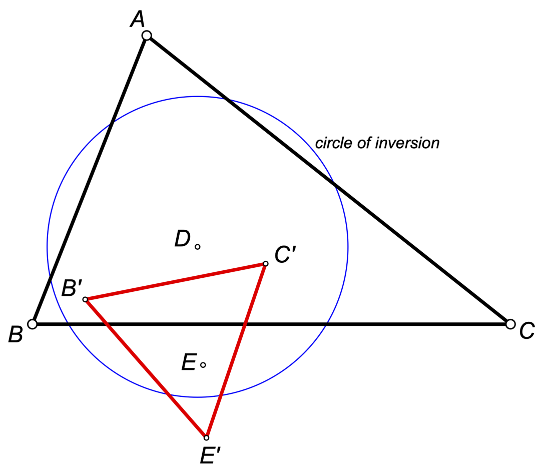


Figure 3. Inverse of $\triangle BCE$ about D

Proof. By Lemma 3.2, D is the 2nd isodynamic point of $\triangle BCE$. The result then follows by Lemma 3.3. □

The following result is well known [6].

Lemma 3.5. *Let ABE, BCF, and CAG be equilateral triangles erected outwardly on the sides of $\triangle ABC$. Then the circumcircles of these equilateral triangles meet at a point (Figure 4). The point of concurrence is the 1st Fermat point (X_{13}) of $\triangle ABC$.*

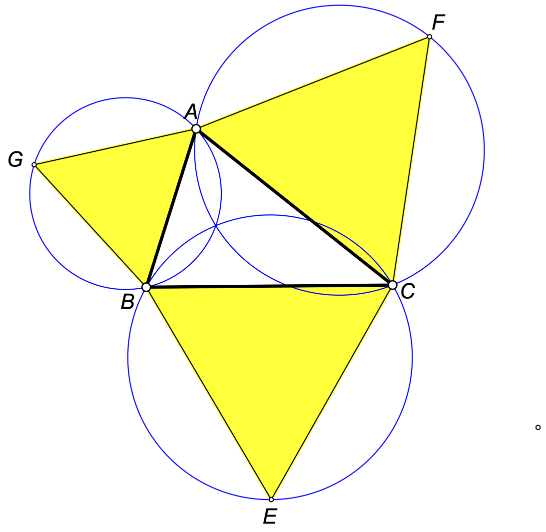


Figure 4. Equilateral triangles erected on the sides of $\triangle ABC$

Lemma 3.6. *Let D be a point inside $\triangle ABC$. Let Γ be the circumcircle of $\triangle BCD$ (Figure 5). Then vertex A lies outside Γ.*

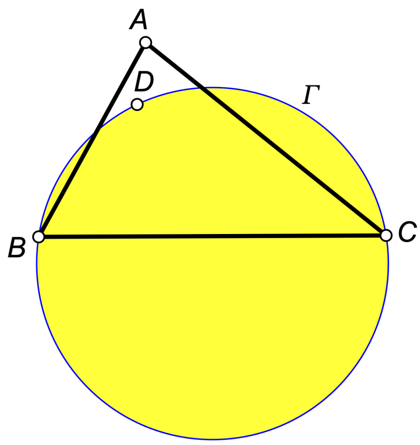


Figure 5.

Proof. Since D is an interior point of $\triangle ABC$, we have $\angle BDC > \angle BAC$. Now Γ is the locus of points X satisfying $\angle BXC = \angle BDC$. Moreover on the side of BC containing D we have $\angle BXC > \angle BDC$ for points inside Γ, and $\angle BXC < \angle BDC$ for points outside Γ. Since $\angle BAC < \angle BDC$, it follows that A lies outside Γ. $\square$

Lemma 3.7. *Let D be any point inside $\triangle ABC$. Let E be the 1st isodynamic point of $\triangle BCD$. Let $\mathbf{I}$ be an inversion with center D and any radius. Let A', B', C', and E' be the images of A, B, C, and E under $\mathbf{I}$, respectively. Then point E' lies on the opposite side of line $B'C'$ from A'.*

Proof. An inversion maps a circle through the center of inversion into a straight line. Points inside that circle get mapped onto one half plane bounded by the straight line and points outside that circle get mapped onto the other half plane.

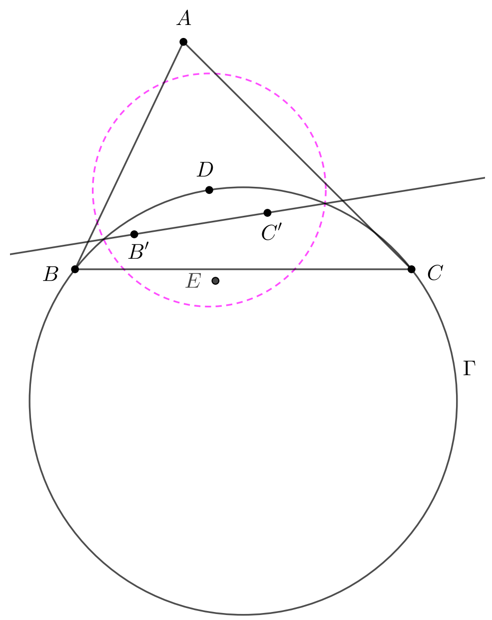


FIGURE 6.

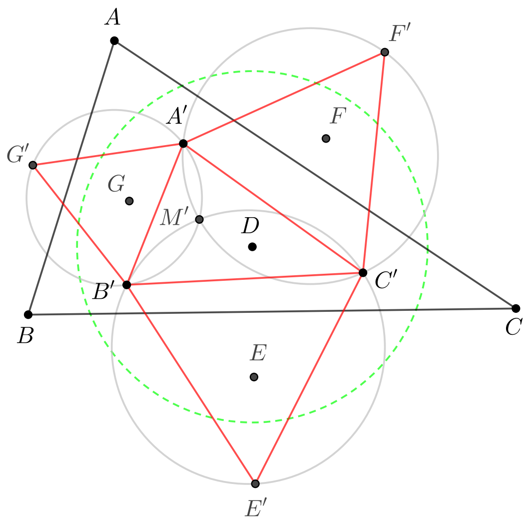
Let Γ be the circumcircle of triangle BCD (Figure 6). Then the inversion maps Γ onto the line $B'C'$. By Lemma 3.1, E lies inside Γ. By Lemma 3.6, A lies outside Γ. Thus, A' and E' lie on opposite sides of $B'C'$. $\square$

4. Proof of Theorem 1.1

Proof. Perform an inversion $\mathbf{I}$ centered at D (with any radius). For a point $P \neq D$, write P' for its image under $\mathbf{I}$.

By Lemma 3.4, $\triangle B'C'E'$ is an equilateral triangle. By Lemma 3.7, A' and E' lie on opposite sides of $B'C'$. Similarly, $C'A'F'$ and $A'B'G'$ are equilateral triangles and the three equilateral triangles are erected outwardly on the sides of $\triangle A'B'C'$.

By Lemma 3.5, circles $\odot B'C'E'$, $\odot C'A'F'$, and $\odot A'B'G'$ meet in a point M'.



Inversion is an involution that carries circles/lines to circles/lines and preserves incidence. Therefore, applying $\mathbf{I}$ back, the circle $B'C'E'$ maps back to the circle (or line) through B, C, and E. The same is true for the other two circles. The common point M' gets mapped back to the single point $\mathbf{I}(M') = M$. Therefore M lies on all three circles $\odot EBC$, $\odot FCA$, and $\odot GAB$. In other words, these three circles concur. Thus, triangles ABC and EFG are cyclologic. □

Note. Theorem 1.1 is not necessarily true if point D lies outside $\triangle ABC$.

5. Related Results

Geometric proofs for the following two results were given in [4].

Theorem 5.1. *Let D be any point inside $\triangle ABC$. Let E, F, and G be the $X(13)$-points of triangles BCD, CAD, and ABD, respectively. Then triangles ABC and EFG are cyclologic.*

Theorem 5.2. *Let D be any point inside $\triangle ABC$. Let E, F, and G be the $X(14)$-points of triangles BCD, CAD, and ABD, respectively. Then triangles ABC and EFG are cyclologic.*

Theorem 5.3. *Let D be any point inside $\triangle ABC$. Let E, F, and G be the $X(16)$-points of triangles BCD, CAD, and ABD, respectively. Then triangles ABC and EFG are cyclologic.*

The proof of this theorem is similar to the proof of Theorem 1.1 and the details are omitted. In this case, an inversion about point D gives three equilateral triangles erected inwardly on $\triangle E'F'G'$.

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