Centers of a Family of Eight-Point Conics Associated with a Cyclic Quadrangle

Stanley Rabinowitz
545 Elm St Unit 1
Milford, New Hampshire 03055, USA
stan.rabinowitz@comcast.net

Abstract

Chomicz, Płatek, Smolira, and Wyrzykowski proved that a triangle center with constant Shinagawa coefficients gives rise, by isogonal conjugation, to an eight-point conic associated with every cyclic quadrangle. We give a short complex-coordinate derivation of this family and an explicit formula for the center of each finite member. As a consequence, we prove a 2025 observation of Miyamoto: the conic through the four vertices of a cyclic quadrangle and the four Lemoine points of its component triangles has center QA-$P44$, the point minimizing the sum of the squared distances to the six sidelines. Moreover, the Lemoine member is the unique finite member of the family whose center is identically QA-$P44$. Generically, the family exhausts the pencil of circumconics, and its centers trace the nine-point conic QA-$Co1$. In particular, QA-$P44$ lies on QA-$Co1$ for every cyclic quadrangle.

Keywords. cyclic quadrangle, Lemoine point, symmedian point, eight-point conic, Shinagawa coefficients, least-squared-distance point, quadrangle center.

Mathematics Subject Classification (2020). 51M04; 51M05, 51M15.

1 Introduction

On January 6, 2025, Dylan Wyrzykowski posed in the Facebook group *Romantics of Geometry* the problem of proving that if $ABCD$ is a cyclic quadrangle and P_A is the Lemoine point $X(6)$ (also known as the symmedian point) of triangle BCD, with P_B, P_C, P_D defined analogously, then

$$A, B, C, D, P_A, P_B, P_C, P_D$$

lie on a conic [14]. Rabinowitz subsequently recorded this problem in the Quadri-Figures Problem Group and asked for a description of the center of the conic [11]. In the same discussion, further examples were recorded and Wyrzykowski conjectured a general connection with triangle centers having constant Shinagawa coefficients.

On January 13, 2025, Miyamoto reported that a GeoGebra experiment indicated that the center of the Lemoine conic coincides with QA-$P44$, the least-squared-distance point of the six sidelines of the quadrangle. He explicitly noted that this identification had not been algebraically confirmed [4]. Related computer-discovered examples were also recorded by Rabinowitz and Suppa [12].

Throughout, $X(n)$ denotes the triangle-center notation of Clark Kimberling's *Encyclopedia of Triangle Centers* (ETC), which we also follow for the definition of Shinagawa coefficients [3].

The general eight-point phenomenon was subsequently proved by Chomicz, Płatek, Smolira, and Wyrzykowski [1]. Let X be a triangle center on the Euler line having constant Shinagawa coefficients

(σ, τ), and let Y be its isogonal conjugate. For a cyclic quadrangle $ABCD$, form the four Y-points of the component triangles BCD, ACD, ABD, ABC. Together with A, B, C, D, these four points lie on a conic. If the circumcircle is normalized to the unit circle and H is the orthocenter of a component triangle, then the corresponding X-point has position vector

$$x = \lambda h, \qquad \lambda = \frac{\sigma + \tau}{3\sigma + \tau}.$$

This parametrization is due to Wyrzykowski [13]. Our derivation below gives, in this setting, a short self-contained proof of the eight-point-conic theorem while simultaneously producing its center.

The Lemoine case corresponds to $X = X(2)$, the centroid, for which $\lambda = 1/3$, and $Y = X(6)$, the Lemoine point. We shall see that Miyamoto's observation is a distinguished member of the general family. We also relate the center formula to Minthorn's classical theorem on centers of circumconics and to the nine-point conic of the quadrangle.

2 The general eight-point conic

By a similarity, put the circumcircle of $ABCD$ on the unit circle in the complex plane, and let the affixes of A, B, C, D be a, b, c, d. Thus

$$|a| = |b| = |c| = |d| = 1.$$

Write

$$\begin{aligned} e_1 &= a + b + c + d, \\ e_2 &= ab + ac + ad + bc + bd + cd, \\ e_3 &= abc + abd + acd + bcd, \\ e_4 &= abcd. \end{aligned} \tag{1}$$

In particular,

$$\bar{e}_1 = \frac{e_3}{e_4}, \qquad \bar{e}_2 = \frac{e_2}{e_4}. \tag{2}$$

For a finite real number λ, consider in each component triangle the point X whose position vector is λh, where h is the position vector of the orthocenter, and let Y be its isogonal conjugate.

Lemma 1. *For triangle BCD, put*

$$s = b + c + d, \qquad t = bc + bd + cd, \qquad r = bcd.$$

If y is the affix of Y_A, the isogonal conjugate of the point with affix $x = \lambda s$, then

$$y = \frac{(1-\lambda)(\lambda t^2 - sr)}{\lambda^2 st - r}, \qquad \bar{y} = \frac{(1-\lambda)(\lambda s^2 - t)}{\lambda^2 st - r}. \tag{3}$$

The formula is understood by continuity in exceptional cases in which the displayed denominator vanishes.

Proof. Since the circumcircle is the unit circle,

$$\bar{s} = \frac{t}{r}, \qquad \bar{x} = \lambda \frac{t}{r}.$$

The standard complex criterion for two points x, y to be isogonal conjugates in triangle BCD is that

$$\frac{(x-b)(y-b)}{(c-b)(d-b)}$$

is real, together with the analogous conditions at c and d. This follows directly by taking arguments and expressing equality of the corresponding directed angles. Writing the conditions at b and c, replacing conjugates of b, c, d by their reciprocals, and solving the resulting two linear equations for $y, \bar{y}$ gives (3). $\square$

Before stating the general conic, we explain how its equation arises. Start with an arbitrary real conic through the four vertices. Apart from the circumcircle itself, we may normalize the coefficient of z^2 to be 1 and write its equation in the form

$$z^2 + Bz\bar{z} + C\bar{z}^2 + Dz + E\bar{z} + F = 0.$$

On the unit circle, $\bar{z} = 1/z$. Multiplying by z^2, the resulting quartic has the four roots a, b, c, d, and hence must equal

$$(z-a)(z-b)(z-c)(z-d) = z^4 - e_1 z^3 + e_2 z^2 - e_3 z + e_4.$$

Comparison of coefficients gives

$$C = e_4, \qquad D = -e_1, \qquad E = -e_3, \qquad B + F = e_2.$$

For a real conic, conjugating its equation and multiplying by e_4 must reproduce the original equation. By (2), this implies

$$e_4 \bar{B} = B, \qquad e_4 \bar{F} = F.$$

When $e_2 \neq 0$, it follows that B/e_2 and F/e_2 are real. Thus, for some real parameter μ, every noncircular real circumconic has the form

$$z^2 + \mu e_2 z\bar{z} + e_4 \bar{z}^2 - e_1 z - e_3 \bar{z} + (1-\mu)e_2 = 0.$$

Substituting the expressions for $y, \bar{y}$ from (3) into this one-parameter equation, its numerator has a factor $\lambda - \mu$; the remaining factor is not identically zero. Hence the member containing the isogonal-conjugate points is obtained by taking $\mu = \lambda$. This is the origin of (4). The exceptional case $e_2 = 0$ follows by continuity and is discussed separately below.

Theorem 2. *For every real λ, the equation*

$$z^2 + \lambda e_2 z\bar{z} + e_4 \bar{z}^2 - e_1 z - e_3 \bar{z} + (1-\lambda)e_2 = 0 \tag{4}$$

defines a real conic containing A, B, C, D and the four points Y_A, Y_B, Y_C, Y_D. Whenever this conic has a finite center, its affix is

$$q_\lambda = \frac{2e_1 e_4 - \lambda e_2 e_3}{4e_4 - \lambda^2 e_2^2}. \tag{5}$$

Proof. First let z be one of a, b, c, d. Since $\bar{z} = 1/z$, multiplication of the left side of (4) by z^2 gives

$$z^4 - e_1 z^3 + e_2 z^2 - e_3 z + e_4 = (z-a)(z-b)(z-c)(z-d).$$

Thus all four vertices lie on the conic.

For Y_A, use (3) together with

$$e_1 = a + s, \qquad e_2 = as + t, \qquad e_3 = at + r, \qquad e_4 = ar.$$

Direct substitution in (4) gives zero identically. By symmetry the same is true for Y_B, Y_C, Y_D. Hence (4) contains the required eight points. Moreover, using (2), multiplication of the conjugate equation by e_4 reproduces (4); hence it is a real conic. For a generic quadrangle the eight points determine this conic uniquely, and exceptional cases are obtained by continuity.

Let q_λ be its center. Replace z by $q_\lambda + w$. The coefficients of the linear terms in $w, \bar{w}$ vanish exactly when

$$2q_\lambda + \lambda e_2 \bar{q}_\lambda = e_1, \qquad \lambda e_2 q_\lambda + 2e_4 \bar{q}_\lambda = e_3. \tag{6}$$

Solving this system gives (5). A finite center exists when

$$4e_4 - \lambda^2 e_2^2 \neq 0.$$

□

Lemma 3. *If a, b, c, d are distinct points of the unit circle, then*

$$|e_2| < 6.$$

Proof. The number e_2 is a sum of six unit complex numbers, so $|e_2| \leq 6$. Equality would force all six products ab, ac, ad, bc, bd, cd to have the same argument. In particular $ab = ac$, which gives $b = c$, a contradiction. □

By (2),

$$\frac{e_2^2}{e_4} = |e_2|^2.$$

Thus Lemma 3 implies that the members $\lambda = 0$ and $\lambda = 1/3$ always have finite centers, and it also implies

$$e_2^2 - 36e_4 = e_4(|e_2|^2 - 36) \neq 0.$$

Remark. The orthocenter member gives a useful check. If $X = X(3) = O$, then $\lambda = 0$ and its isogonal conjugate is $Y = X(4) = H$. Formula (5) becomes

$$q_0 = \frac{e_1}{2}.$$

Indeed, the orthocenter of BCD has affix $e_1 - a$, so A and H_A are centrally symmetric about $e_1/2$, and similarly for the other three vertex–orthocenter pairs. This conic is the unique orthogonal hyperbola *QA-Co*2 through the four vertices, and its center is the Euler–Poncelet point *QA-P*2 [6, 8].

3 The least-squared-distance point

The point *QA-P*44 is characterized in the Encyclopedia of Poly Geometry as the point minimizing the sum of the squared distances to the six sidelines of the quadrangle [10]. We now express this condition in the same normalization.

If u, v are two points of the unit circle, their chord has equation

$$L_{uv}(z) = z + uv\bar{z} - u - v = 0,$$

and the squared Euclidean distance from z to this line is

$$d_{uv}(z)^2 = \frac{L_{uv}(z)^2}{4uv}.$$

Consequently, if

$$D(z) = \sum_{\{u,v\}\subset\{a,b,c,d\}} d_{uv}(z)^2,$$

then, for some real constant C,

$$\begin{aligned} 4D(z) = {} & \frac{e_2}{e_4} z^2 + 12 z\bar{z} + e_2 \bar{z}^2 \\ & - 6\frac{e_3}{e_4} z - 6e_1 \bar{z} + C. \end{aligned} \tag{7}$$

Proposition 4. *If q_{44} is the affix of QA-P44, then*

$$6q_{44} + e_2 \bar{q}_{44} = 3e_1. \tag{8}$$

Equivalently,

$$q_{44} = \frac{3(e_2e_3 - 6e_1e_4)}{e_2^2 - 36e_4}. \tag{9}$$

Proof. As a function of the two real coordinates of z, D is a sum of squares of six affine linear functions. Its quadratic part is a sum of squares of the corresponding unit normals. Since the six sidelines are not all parallel, this quadratic form is positive definite. Hence D has a unique critical point, and that point is its minimum.

At the minimum, the first variation of (7) vanishes. The coefficient of $\bar{w}$ after putting $z = q_{44} + w$ is

$$12q_{44} + 2e_2\bar{q}_{44} - 6e_1,$$

which gives (8). Together with its conjugate equation, this yields (9). The denominator is nonzero by Lemma 3. □

We can now compare the two center conditions directly.

Theorem 5. *Let $ABCD$ be a cyclic quadrangle. For each vertex, let P_A be the Lemoine point of the opposite triangle BCD, and define P_B, P_C, P_D analogously. Then the conic through*

$$A, B, C, D, P_A, P_B, P_C, P_D$$

has a finite center, and that center is QA-P44.

Proof. The centroid $X(2)$ is the point $x = \frac{1}{3}h$ on the Euler line, so here $\lambda = 1/3$. Its isogonal conjugate is the Lemoine point $X(6)$. Lemma 3 shows that this member has a finite center. Its center satisfies

$$2q_{1/3} + \frac{1}{3}e_2\bar{q}_{1/3} = e_1,$$

or, equivalently,

$$6q_{1/3} + e_2\bar{q}_{1/3} = 3e_1.$$

By Proposition 4, this is precisely the normal equation for QA-P44. □

The comparison gives a little more.

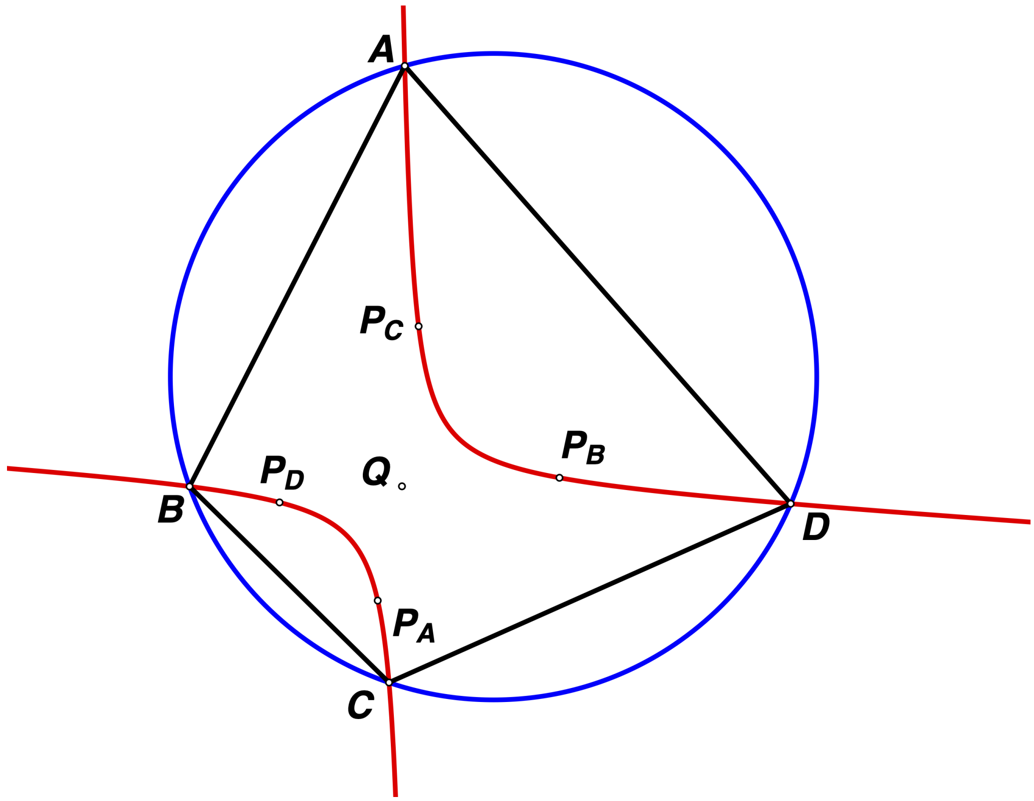

Figure 1: The Lemoine eight-point conic has center $Q = QA\text{-}P44$.

Theorem 6. *Among the finite members of the Chomicz–Płatek–Smolira–Wyrzykowski family, the Lemoine member is the unique one whose center is identically QA-$P44$ for every cyclic quadrangle.*

Proof. If the center q_λ of a fixed finite member were always q_{44}, then (6) and (8) would give

$$\left(\lambda - \frac{1}{3}\right) e_2 \bar{q}_{44} = 0$$

for every cyclic quadrangle. The factor $e_2\bar{q}_{44}$ is not identically zero; for example, direct substitution of

$$(a, b, c, d) = (1, i, -1, e^{i\pi/3})$$

gives $e_2\bar{q}_{44} \neq 0$. Hence $\lambda = 1/3$. Conversely, Theorem 5 shows that $\lambda = 1/3$ has the required property. □

4 The nine-point conic

Minthorn's classical theorem concerns the centers of conics through the four vertices of a quadrangle. In the nontrapezoidal, nondegenerate case, those centers trace the nine-point conic. See Kaldybayev [2] for a modern elementary proof. In the notation of the Encyclopedia of Poly Geometry, the nine-point conic is QA-$Co1$ [5]. We shall identify it directly in our coordinates, so that no converse form of Minthorn's theorem is needed in the exceptional cases.

Let $F_\lambda(z, \bar{z})$ denote the left side of (4). Then

$$F_\lambda(z, \bar{z}) = F_0(z, \bar{z}) + \lambda e_2(z\bar{z} - 1). \tag{10}$$

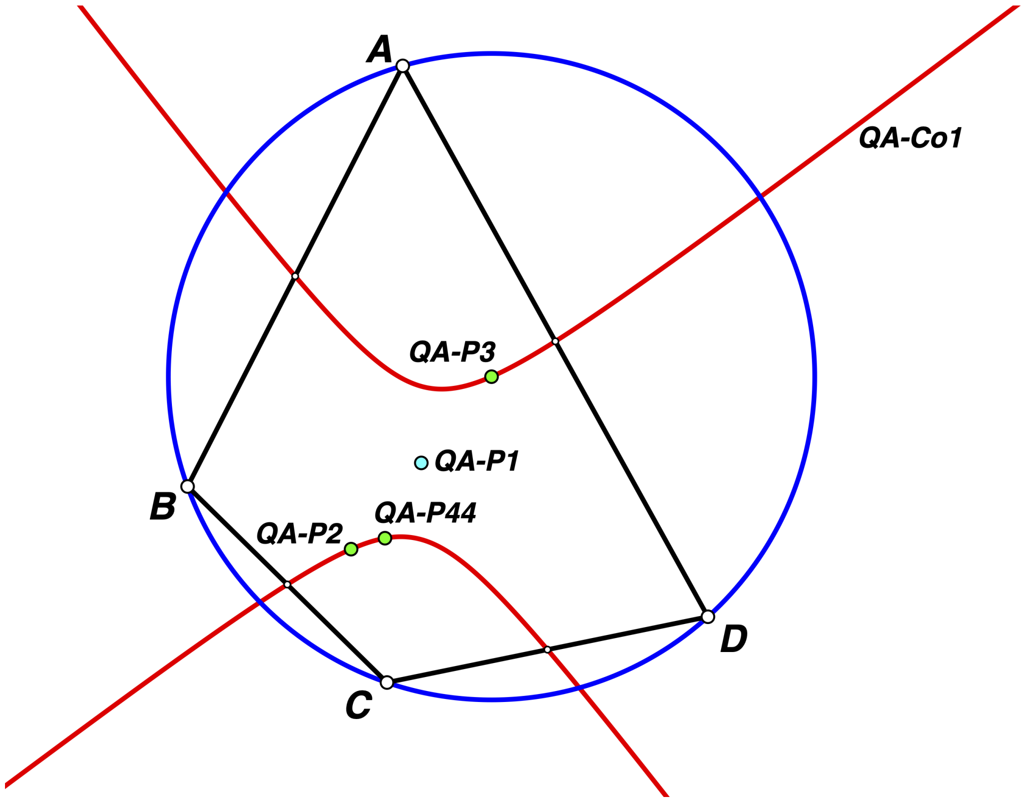

Figure 2: The centers $QA\text{-}P2$, $QA\text{-}P44$, and $QA\text{-}P3$ lie on the nine-point conic $QA\text{-}Co1$, whose center is $QA\text{-}P1$.

Thus, if $e_2 \neq 0$, the projective parameter $\lambda \in \mathbb{R} \cup \{\infty\}$ runs through the entire pencil of conics through A, B, C, D, with $\lambda = \infty$ corresponding to the circumcircle $z\bar{z} = 1$.

Theorem 7. *Every finite center q_λ of a conic* (4) *satisfies*

$$2z^2 - e_1 z + e_3 \bar{z} - 2e_4 \bar{z}^2 = 0. \tag{11}$$

Equation (11) *is the nine-point conic $QA\text{-}Co1$, and its center is*

$$\frac{e_1}{4} = QA\text{-}P1.$$

After translation to its center it takes the form

$$2w^2 - 2e_4 \bar{w}^2 + \kappa = 0, \qquad \kappa = -\frac{(ab - cd)(ac - bd)(ad - bc)}{8e_4}. \tag{12}$$

Consequently, $QA\text{-}Co1$ is a nondegenerate rectangular hyperbola if and only if

$$(ab - cd)(ac - bd)(ad - bc) \neq 0. \tag{13}$$

If $e_2 \neq 0$ and (13) *holds, then as λ varies projectively, the centers of the family trace the whole nine-point conic.*

Proof. Multiply the first equation of (6) by q_λ, the second by $\bar{q}_\lambda$, and subtract. This eliminates λe_2 without division and gives

$$2q_\lambda^2 - e_1 q_\lambda + e_3\bar{q}_\lambda - 2e_4\bar{q}_\lambda^2 = 0.$$

Hence every finite center lies on (11).

We next identify this conic directly. For the midpoint $m = (a+b)/2$, we have $\bar{m} = (a+b)/(2ab)$. Substitution in the left side of (11) reduces to

$$\frac{a+b}{2}\left((a+b) - e_1 + \frac{e_3}{ab} - \frac{cd(a+b)}{ab}\right) = 0,$$

because

$$\frac{e_3}{ab} = c + d + \frac{cd(a+b)}{ab}.$$

Thus all six pairwise midpoints lie on (11). If $x = AB \cap CD$ and $ab \neq cd$, then the two chord equations give

$$\bar{x} = \frac{a+b-c-d}{ab-cd}, \qquad x = \frac{ab(c+d) - cd(a+b)}{ab-cd},$$

and direct substitution in (11) again gives zero. The other two diagonal points follow by symmetry. When one of the relevant pairs of chords is parallel, the corresponding diagonal point is at infinity and the same statement follows projectively, or by continuity. Therefore (11) contains the six midpoints and the three diagonal points that define the nine-point conic $QA\text{-}Co1$.

Translate $z = e_1/4 + w$. Using $e_3 = e_4\bar{e}_1$, equation (11) becomes

$$2w^2 - 2e_4\bar{w}^2 + \kappa = 0,$$

where

$$\kappa = \frac{e_3^2 - e_1^2 e_4}{8e_4} = -\frac{(ab-cd)(ac-bd)(ad-bc)}{8e_4}.$$

The second equality is an elementary identity in a, b, c, d. Thus $e_1/4$, the arithmetic mean of the four vertices, is the center $QA\text{-}P1$. Since the quadratic part has no $z\bar{z}$ term, the nondegenerate conic is rectangular. It degenerates into two perpendicular lines exactly when $\kappa = 0$, which is precisely the failure of (13).

Finally, suppose $e_2 \neq 0$ and (13) holds. Equation (10) shows that our family is the entire pencil of circumconics, while the nondegenerate form of Minthorn's theorem shows that their centers trace the whole nine-point conic. This completes the proof. □

Remark. The condition $\kappa = 0$ has a simple geometric meaning. The equality $ab = cd$ is equivalent to $AB \parallel CD$, and similarly for the other two factors. Thus degeneracy occurs exactly when one of the three opposite pairs in the complete quadrangle,

$$AB, CD; \qquad AC, BD; \qquad AD, BC,$$

is parallel. If A, B, C, D are listed in cyclic order, this is precisely the isosceles-trapezoid case (rectangles included). Then $QA\text{-}Co1$ is a pair of perpendicular lines through $QA\text{-}P1$; the centers from our family lie on only one of the two components, the symmetry axis. In the rectangle subcase $e_1 = 0$, all nonsingular central members have center

$$QA\text{-}P1 = QA\text{-}P2 = QA\text{-}P3 = QA\text{-}P44 = 0.$$

Corollary 8. *For every cyclic quadrangle,*

$$QA\text{-}P44 \in QA\text{-}Co1.$$

Proof. By Theorem 5, QA-$P44$ is the finite center of the Lemoine member of the family. Theorem 7 therefore puts it on QA-$Co1$, whether or not that conic is degenerate. □

Remark. Three members of the pencil give especially familiar points of QA-$Co1$. The value $\lambda = 0$ gives the orthocenter member and center QA-$P2$. The value $\lambda = 1/3$ gives the Lemoine member and center QA-$P44$. Finally, $\lambda = \infty$ gives the circumcircle, whose center is the origin in our normalization. For a cyclic quadrangle this point is QA-$P3$: the Gergonne–Steiner conic has least eccentricity among the circumconics, and in the cyclic case the least possible eccentricity is 0 [9]. Thus we recover the known relation that QA-$P1$ is the midpoint of QA-$P2$ and QA-$P3$ [5].

Remark. The denominator in (5) vanishes when

$$4e_4 - \lambda^2 e_2^2 = 0.$$

Since $e_2^2/e_4 = |e_2|^2$, when $e_2 \neq 0$ the two values are

$$\lambda = \pm\frac{2}{|e_2|}.$$

If QA-$Co1$ is nondegenerate, these are the two circumscribed parabolic members of the pencil, whose centers are the two points at infinity of QA-$Co1$, in agreement with the classical relation between the nine-point conic and the pair of circumscribed quadrangle parabolas [7]. In the degenerate trapezoid case, one of these singular parameter values may instead give a degenerate pair of parallel lines.

Remark. The case $e_2 = 0$ is exceptional. Equation (4) then becomes independent of λ, so the constant-Shinagawa construction collapses to a single conic instead of parametrizing the full pencil. Formula (5) gives $q_\lambda = e_1/2$ for every finite λ; in particular, Proposition 4 shows that QA-$P44 = QA$-$P2$ in this case.

Acknowledgments

The author thanks Keita Miyamoto for his experimental discovery that the center of the Lemoine eight-point conic is QA-$P44$, and for helpful comments on the manuscript. He thanks Dylan Wyrzykowski for posing the original Lemoine eight-point-conic problem, for recognizing its connection with constant Shinagawa coefficients, and for his subsequent work on the general family. He also thanks Chris van Tienhoven, a cofounder of the *Encyclopedia of Poly Geometry* (EPG), for maintaining this valuable resource and for discovering a geometric construction of QA-$P44$. Finally, the author thanks ChatGPT, developed by OpenAI, for substantial assistance in discovering the general center formula and the nine-point-conic locus, carrying out and checking symbolic algebra, locating relevant literature, and organizing the proof. The author is responsible for the final mathematical statements and presentation.

References

[1] Kazimierz Chomicz, Miłosz Płatek, Konstanty Smolira, and Dylan Wyrzykowski, *A Family of Eight-Point Conics Associated with the Cyclic Quadrilateral*, Journal for Geometry and

Graphics **29** (2025), no. 2, 173–185.
https://www.heldermann.de/JGG/JGG29/JGG292/jgg29018.htm
https://arxiv.org/abs/2511.13298

[2] Rauan Kaldybayev, *A Generalization of Newton's Quadrilateral Theorem and an Elementary Proof of Minthorn's Quadrilateral Theorem*, arXiv:2211.09764, 2022.
https://arxiv.org/abs/2211.09764

[3] Clark Kimberling, *Encyclopedia of Triangle Centers (ETC)*, with supporting tables and material on Shinagawa coefficients.
https://faculty.evansville.edu/ck6/encyclopedia/etc.html
https://faculty.evansville.edu/ck6/encyclopedia/Tables.html
https://faculty.evansville.edu/ck6/encyclopedia/Euler%20Coordinates.pdf

[4] Keita Miyamoto, *Re: Two quadrangles with a common circumconic*, QPG Journal 2025, Message #2566, January 13, 2025.
https://www.chrisvantienhoven.nl/wp-content/themes/astra-child/epg-downloads/QPG-Journals/QPG-Journal-2025.pdf

[5] Chris van Tienhoven, *QA-Co1: Nine-point Conic*, Encyclopedia of Poly Geometry.
https://www.chrisvantienhoven.nl/epg/n-geometry/quadri-figures/qa-items/qa-co1/

[6] Chris van Tienhoven, *QA-Co2: QA-Orthogonal Hyperbola*, Encyclopedia of Poly Geometry.
https://www.chrisvantienhoven.nl/epg/n-geometry/quadri-figures/qa-items/qa-co2/

[7] Chris van Tienhoven, *QA-2Co1: Pair of Circumscribed QA-Parabolas*, Encyclopedia of Poly Geometry.
https://www.chrisvantienhoven.nl/epg/n-geometry/quadri-figures/qa-items/qa-2co1/

[8] Chris van Tienhoven, *QA-P2: Euler-Poncelet Point*, Encyclopedia of Poly Geometry.
https://www.chrisvantienhoven.nl/epg/n-geometry/quadri-figures/qa-items/qa-p2/

[9] Chris van Tienhoven, *QA-P3: Gergonne-Steiner Point*, Encyclopedia of Poly Geometry.
https://www.chrisvantienhoven.nl/epg/n-geometry/quadri-figures/qa-items/qa-p3/

[10] Chris van Tienhoven, *QA-P44: Least Squared Distance Point of QA-sidelines*, Encyclopedia of Poly Geometry.
https://www.chrisvantienhoven.nl/epg/n-geometry/quadri-figures/qa-items/qa-p44/

[11] Stanley Rabinowitz, *Two quadrangles with a common circumconic*, QPG Journal 2025, Message #2563, January 13, 2025.
https://www.chrisvantienhoven.nl/wp-content/themes/astra-child/epg-downloads/QPG-Journals/QPG-Journal-2025.pdf

[12] Stanley Rabinowitz and Ercole Suppa, *More Relationships between a Central Quadrilateral and its Reference Quadrilateral*, International Journal of Computer Discovered Mathematics **10**

(2025), 91–123; arXiv:2506.17240.
https://arxiv.org/abs/2506.17240

[13] Dylan Wyrzykowski, *Cycling along Euler road*, Journal of Geometry **116** (2025), Article 43.
https://doi.org/10.1007/s00022-025-00782-5

[14] Dylan Wyrzykowski, post in the Facebook group *Romantics of Geometry*, January 6, 2025.
https://www.facebook.com/groups/parmenides52/posts/9041609412619351