# The Tripolar Coordinates for the Electrostatic Center of a Triangle

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**Abstract**

The electrostatic center of a triangle is the point at which the electrostatic potential generated by a uniform charge on the triangular lamina is maximal. Abraham and Kovač gave an implicit characterization of this point in terms of its distances from the vertices. We observe that their formulas immediately yield a particularly simple expression for the tripolar coordinates of the electrostatic center.

**Keywords.** electrostatic center, tripolar coordinates, triangle centers.

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## 1 Introduction

Let $ABC$ be a triangle with side lengths

$$a = BC, \qquad b = CA, \qquad c = AB.$$

The *electrostatic center* of $ABC$, now listed as $X(5626)$ in Kimberling's *Encyclopedia of Triangle Centers* (ETC), was introduced by Hrvoje Abraham and Vjekoslav Kovač [1]. It is the unique point $P$ at which the electrostatic potential generated by a uniform charge distributed over the triangular lamina is maximal.

Following the convention used for tripolar coordinates, the homogeneous *tripolar coordinates* of a point $P$ are

$$PA : PB : PC.$$

Thus a formula for the three vertex distances of the electrostatic center gives its tripolar coordinates at once. The purpose of this note is to record the resulting formula explicitly, using the notation of the ETC entry for $X(5626)$ [2].

## 2 Tripolar coordinates

We use the notation appearing in the ETC entry for $X(5626)$. Set

$$g_a = \coth \frac{a\lambda}{a+b+c}, \qquad g_b = \coth \frac{b\lambda}{a+b+c}, \qquad g_c = \coth \frac{c\lambda}{a+b+c},$$

and

$$u = ag_a, \qquad v = bg_b, \qquad w = cg_c,$$

where $\lambda$ is the unique positive solution of

$$\begin{aligned}\sqrt{(u^2-a^2)\big(a^2-(v-w)^2\big)}+\sqrt{(v^2-b^2)\big(b^2-(w-u)^2\big)}\\ +\sqrt{(w^2-c^2)\big(c^2-(u-v)^2\big)}\\ =\sqrt{2(b^2c^2+c^2a^2+a^2b^2)-a^4-b^4-c^4}.\end{aligned} \tag{1}$$

If $q$ denotes the Abraham–Kovač parameter in the form

$$PB+PC=a\coth(aq),$$

then

$$q=\frac{\lambda}{a+b+c}.$$

Thus $g_a=\coth(aq)$, and cyclically.

With this notation, the distance characterization of Abraham and Kovač may be written

$$PB+PC=u,\qquad PC+PA=v,\qquad PA+PB=w. \tag{2}$$

The tripolar coordinates now take a particularly simple form.

**Theorem 1.** *Let $P=X(5626)$. Then its distances from the vertices are*

$$PA=\frac{v+w-u}{2},\qquad PB=\frac{w+u-v}{2},\qquad PC=\frac{u+v-w}{2}.$$

*Consequently, the tripolar coordinates of $X(5626)$ are*

$$\big(v+w-u:w+u-v:u+v-w\big),$$

*where*

$$g_a=\coth\frac{a\lambda}{a+b+c},\qquad g_b=\coth\frac{b\lambda}{a+b+c},\qquad g_c=\coth\frac{c\lambda}{a+b+c},$$
$$u=ag_a,\qquad v=bg_b,\qquad w=cg_c,$$

*and $\lambda$ is the unique positive solution of* (1).

*Proof.* The three distance formulas follow immediately by solving the linear system (2); for example, adding the second and third equations and subtracting the first gives $2PA=v+w-u$. The other two follow cyclically. Since tripolar coordinates are homogeneous, the common factor $1/2$ may be omitted. $\square$

Thus the notation already used in the ETC entry for the trilinear coordinates of $X(5626)$ gives the tripolar coordinates in the compact form

$$(v+w-u:w+u-v:u+v-w).$$

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The author is responsible for the final mathematical statements and presentation.

## References


[1] Hrvoje Abraham and Vjekoslav Kovač, *From electrostatic potentials to yet another triangle center*, Forum Geometricorum **15** (2015), 73–89.

[2] Clark Kimberling, *Encyclopedia of Triangle Centers*, `https://faculty.evansville.edu/ck6/encyclopedia/ETC.html`.