Sangaku Journal of Mathematics (SJM) ©SJM
ISSN 2534-9562
Volume 10 (2026), pp. 67–68
Received 16 October 2026. Published on-line 17 October 2026
web: http://www.sangaku-journal.com/
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Four Congruent Circles in a Right Triangle

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Abstract. Three congruent circles form a tangent chain along the hypotenuse of a right triangle, with the end circles tangent to the respective legs. We show that a fourth circle inside the triangle, tangent to both legs and externally tangent to the middle circle, is necessarily congruent to the original three circles.

Keywords. tangent circles, congruent circles, circle chain, right triangle, sangaku.

Mathematics Subject Classification (2020). 51M04.

1. Introduction

Chains of tangent circles have long interested Japanese geometers. Two problems in the historical collection *Kioshu*, available through the Tohoku University Digital Archives (ToUDA) [2], are particularly close to the present arrangement. File number 200 shows four congruent circles in a right triangle, three of them forming a chain along one leg. File number 95 shows five congruent circles in a right triangle, with four forming a chain along the hypotenuse and the fifth near the right-angle vertex. Okumura [1] also studied configurations of congruent circles tangent to a common line and noted that one involving five circles appears in a problem proposed by Shinohara in 1809 (see his Figure 6). These examples suggest looking for further properties of congruent-circle chains.

Suppose that three congruent circles are tangent to the hypotenuse of a right triangle, with consecutive circles tangent and the two end circles tangent to the respective legs. If another circle is placed inside the triangle near the right-angle vertex so as to be tangent to both legs and externally tangent to the middle circle, then this fourth circle turns out to be congruent to the original three circles.

The result is elementary, but pleasing. We give a short geometric proof.

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2. The theorem

Theorem 2.1. *Let ABC be a right triangle with $\angle A = 90^\circ$. Let C_1, C_2, C_3 be three congruent circles of radius r inside $\triangle ABC$, each tangent to the hypotenuse BC. Assume that C_1 is tangent to AB, that C_2 is tangent to C_1 and C_3, and that C_3 is tangent to AC. Suppose that a fourth circle C_4 lies inside the triangle near A, is tangent to AB and AC, and is externally tangent to C_2. Then C_4 also has radius r.*

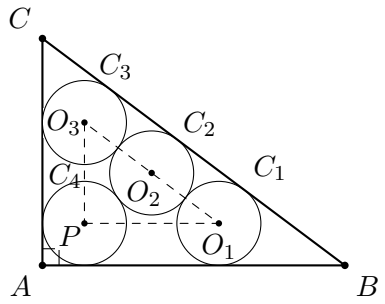


FIGURE 1. The four circles and the auxiliary right triangle O_1PO_3 (dashed).

Proof. Let O_1, O_2, and O_3 be the centers of C_1, C_2, and C_3, respectively. Construct inside $\angle BAC$ a circle of radius r tangent to both legs AB and AC, and let its center be P.

Because this circle and C_1 have the same radius and are tangent to AB from the same side, the segment PO_1 is parallel to AB. Similarly, PO_3 is parallel to AC. Hence

$$\angle O_1PO_3 = \angle BAC = 90^\circ.$$

Since all three circles touch BC and have the same radius, their centers are collinear. Consecutive tangencies give

$$O_1O_2 = O_2O_3 = 2r.$$

Thus O_2 is the midpoint of O_1O_3. Since the midpoint of the hypotenuse of a right triangle is equidistant from all three vertices,

$$PO_2 = O_1O_2 = O_2O_3 = 2r.$$

Hence the circle of radius r centered at P is externally tangent to C_2, as well as tangent to both legs. Since it is the circle next to A with these tangencies, it is C_4. □

Acknowledgment. The author gratefully acknowledges assistance from ChatGPT (OpenAI) in the exploration of the problem and its proof. Responsibility for the final arguments rests with the author.

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