THE INTERCOLLEGIATE MATHEMATICS LEAGUE

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During the 1965–66 school year, a group of college students in New York City banded together to form the Intercollegiate Mathematics League (ICML). The purpose of the League is to coordinate mathematical competitions among its member schools and by so doing to stimulate the students of these schools with the interesting problems proposed.

The problems used are selected by the Faculty Advisor, presently Professor Salkind of the Polytechnic Institute of Brooklyn, from those contributed by volunteers from the faculty of the participating colleges. The contests are an hour long and are taken by teams from each member school. A team consists of from five to ten undergraduates and the team's score is the sum of their top five scores. Both individual and team prizes are awarded.

Because of a late start, only three contests were held during the 1965–66 year. They are listed in the appendix. For this year, the winning school was Cooper Union. Second and third places went to Polytechnic Institute of Brooklyn and Adelphi University, respectively. The highest individual scorer was Harry Ploss of Cooper Union and second and third place went to Stanley Rabinowitz and Richard Lary both of Polytechnic Institute of Brooklyn. The schools received trophies for their achievement and the individuals were given books on mathematics.

The schools which are presently in the League are Adelphi University, Brooklyn College, The Cooper Union, Iona College, Newark College of Engineering, New York Institute of Technology, New York University (at Washington Square and University Heights), Polytechnic Institute of Brooklyn, Pratt Institute, Queens College, and St. John's University. Other schools in or around the Metropolitan New York area which are interested may join by contacting the president of the League, Mark Brody, at Cooper Union or the author.

Appendix. The following are the first three contests held by the ICML.

Meet I

1. Find all the positive values of a for which $\sum_{n=1}^{\infty}(a^n n!)/n^n$ converges.

2. Let a be a given positive number. For each positive number, b, the graph of $x^2/a^2+y^2/b^2=1$ is an ellipse (or a circle). Let the line with equation $x=x_1$, $0<x_1<a$, intersect each of these graphs at two points. Prove that the tangents to the graphs at the intersection points all meet the x-axis in the same point.

3. Prove that if r_1 and r_2 are the roots of $x^2-6x+1=0$, then $r_1^n+r_2^n$, n a natural number, is an integer.

4. Prove that triangles having vertices at lattice points in the xy-plane cannot be equilateral. (A lattice point is one with integer coordinates.)

5. Let $x_1+x_2+\cdots+x_n=s$ where $x_i>0$, $i=1,2,\ldots,n$ and s is fixed. Prove (a) If $p=x_1x_2\cdots x_n$, then p (maximum) $=s^n/n^n$.

(b) $\sqrt[n]{(x_1x_2\ldots x_n)}\le(x_1+x_2+\cdots+x_n)/n$, that is, the positive geometric mean of n positive numbers can never exceed their arithmetic mean.

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Meet II

1. Prove, by the use of "Taylor's Theorem with Remainder" that any curve with equation $y = f(x)$, having an inflection point at $x = x_0$, and for which the second derivative is continuous, crosses the tangent line at x_0. [Define an inflection point as one where the second derivative changes sign.]

2. Find the minimum of the function F where $F(\lambda) = \int_0^1 \left[x^2 - (x + \lambda)\right]^2 dx$.

3. Find the set S of points $P(x, y)$ such that P is equidistant from the x-axis and the circle with center at the origin and radius a.

4. (a) Find the points x_1 and x_2 so that the formula $\int_0^1 f(x)dx = f(x_1) + f'(x_2)$ yields exact results for polynomials of degree ≤ 2. (You may assume f possesses the properties you deem necessary for this problem.) (b) Determine the error (in absolute value) in using this formula for any third-degree polynomial with leading coefficient 1.

5. Define A_n recursively by $A_{n+1} = A_n^2 - A_n + 1$ with $A_1 = 2$. Prove that each A_i is relatively prime to all other A_j, $i \neq j$, $i, j = 1, 2, 3, \ldots$.

Meet III

1. Find the $\lim_{x\to\infty}[x]/x$, where $[x]$ is the greatest integer less than or equal to x. Show all work.

2. Prove or disprove that you can add a constant to the variable of integration if you subtract it from the limits of integration.

3. Derive the general term $u_n(x)$ of the series for

$$\frac{1}{1-x} \ln \frac{1}{1-x} .$$

4. Prove without using l'Hôpital's Rule that $I_1 \approx I_2$ when s is near 1, where

$$I_1 = \int_0^1 \frac{dx}{(1+x)^s} \quad \text{and} \quad I_2 = \int_0^1 \frac{dx}{(1+x)} .$$

5. Let f be a family of lines such that for each line, OP is the x-intercept and OQ is the y-intercept and segment PQ is constant, that is, $PQ = k \neq 0$, k constant. Find the equation of the envelope of f.

DEFINITION. The envelope of a family f of ∞^1 curves is the curve (or curves) such that, at each of its points, it is tangent to a member of the given family f.