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A Divisibility Problem

E 1918 [1966, 891]. *Proposed by L. J. Warren and Jerry Tice, San Diego State College*

Let p be a prime larger than 3. Show that there is no positive integer k such that $3p \mid \sigma_k(3p)$, where $\sigma_k(n)$ is the sum of the kth powers of the divisors of n.

Solution by Stanley Rabinowitz, Far Rockaway, N. Y. The only divisors of $3p$ are 1, 3, p, and $3p$. Now $3p \mid (1+3^k+p^k+3^kp^k)$ implies $3 \mid p^k+1$, which implies $p \equiv 2 \pmod 3$ and k is odd. If $p \mid 3^k+1$, then we would have $3^{k+1} \equiv -3 \pmod p$ with $k+1$ being even, but this contradicts the known fact that if $p \equiv 2 \pmod 3$ then -3 is a quadratic nonresidue (mod p). See problem 3721 [1936, 583].