Reprinted from The American Mathematical Monthly 75.3(1968)296.

Sum of Digits of a Number

E 1926 [1966, 1016]. *Proposed by L. D. Yarbrough, Harvard Computing Center*

Express in terms of N and b the sum of the digits of the integer N as written in radix b notation. (This is a generalization of the rule of "casting out nines," and for $b=2$ the formula yields the number of 1's in the binary representation of N, which is a measure of the multiplication speed of certain digital computers.)

Solution by Stanley Rabinowitz, Far Rockaway, N. Y. Suppose $N=\sum_{k=0}^{n} a_k b^k$. Then

$$a_j = \left[\frac{N}{b^j}\right] - b\left[\frac{N}{b^{j+1}}\right],$$

so

$$\begin{aligned}\sum_{j=0}^{n} a_j &= \sum_{j=0}^{\infty}\left(\left[\frac{N}{b^j}\right] - b\left[\frac{N}{b^{j+1}}\right]\right)\\ &= \sum_{j=0}^{\infty}\left[\frac{N}{b^j}\right] - b\sum_{j=1}^{\infty}\left[\frac{N}{b^j}\right] = N-(b-1)\sum_{j=1}^{\infty}\left[\frac{N}{b^j}\right].\end{aligned}$$