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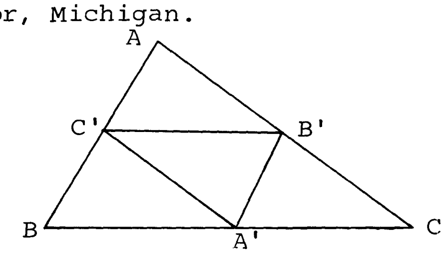
178. Proposed by K. S. Murray, Ann Arbor, Michigan.
Show that the centroid of triangle ABC coincides with that of triangle A'B'C' where A', B', and C' are the midpoints of $\overline{BC}$, $\overline{CA}$, and $\overline{AB}$, respectively. Also, generalize the result.



Solution by Stanley Rabinowitz, Polytechnic Institute of Brooklyn.

Since AB'A'C' is a parallelogram, AA' bisects B'C'. Hence AA' is a median of both triangle ABC and A'B'C'. Hence the medians of both these triangles meet at the same point.

Generalization: Let $A_o, A_1, A_2, \ldots, A_r$ be the vertices of an r-simplex and let B_i be the centroid of the (r-1)-dimensional face opposite A_i, i = 0, 1, ..., r. Then the centroid of the r-simplex with vertices $B_o, \ldots, B_r$ is the same as the centroid of the original r-simplex.

Proof: We use the following facts. The medians of an r-simplex meet at the centroid and this point is 1/(r+1) of the way up from the base. [A median of an r-simplex is a line going from a vertex to the centroid of the opposite face.] Therefore points $B_1', B_2', \ldots, B_r'$ form an r-simplex homothetic to the original one. Therefore median B_oB_o' is also a median of the medial r-simplex since it passes through the centroid of the r-simplex formed by $B_o, B_1', B_2', \ldots, B_r'$. So both sets of medians meet at the same point. Hence the r-simplex and its medial simplex have the same centroid.