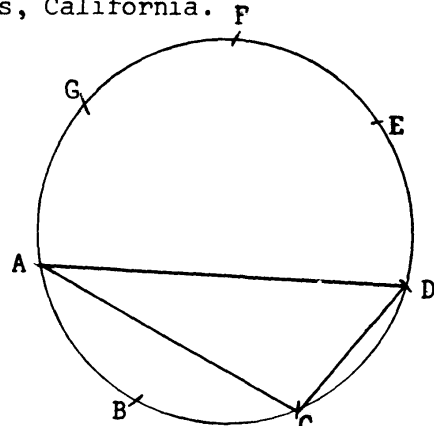


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189. Proposed by Leon Bankoff, Los Angeles, California. F
 If A,B,C,D,E,F, and G denote the consecutive vertices of a regular heptagon, show that CD is equal to half the harmonic mean of AC and AD.



Solution by Stanley Rabinowitz, Polytechnic Institute of Brooklyn. By the law of sines, CD, AC, and AD are proportional to $\sin \pi/7$, $\sin 2\pi/7$, and $\sin 3\pi/7$. Since

$$\begin{aligned} \frac{1}{\sin 2\pi/7} + \frac{1}{\sin 3\pi/7} &= \frac{\sin 2\pi/7 + \sin 3\pi/7}{\sin 2\pi/7 \sin 3\pi/7} = \frac{\sin 2\pi/7 + \sin 3\pi/7}{2 \sin \pi/7 \cos \pi/7 \sin 3\pi/7} \\ &= \frac{\sin 2\pi/7 + \sin 3\pi/7}{\sin 2\pi/7 + \sin 4\pi/7} \cdot \frac{1}{\sin \pi/7} = \frac{1}{\sin \pi/7}, \end{aligned}$$

$$\frac{1}{CD} = \frac{1}{AC} + \frac{1}{AD} \quad \text{which was to be shown.}$$