

figure number 184 of 665

$F = \text{equilateralTriangleCCW}[BC]$

level 2.2

points: {A, B, C, D, E, F}

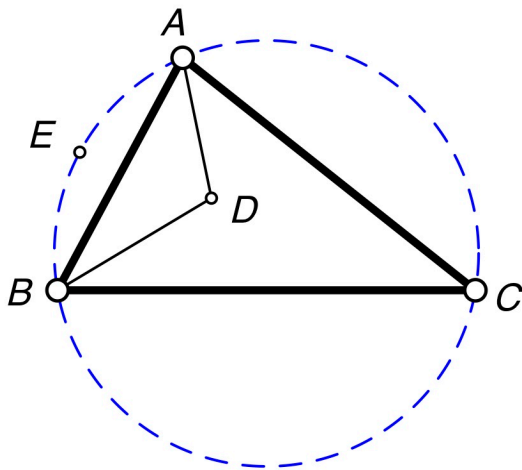
lines: {AB, BC, CD, AD, AE, BE, BF, CF}

triangles: {ABE, BCF}

$\{ABCD = \text{square}[A, B, C, D], E = \text{equilateralTriangleCW}[AB], F = \text{equilateralTriangleCCW}[BC]\}$

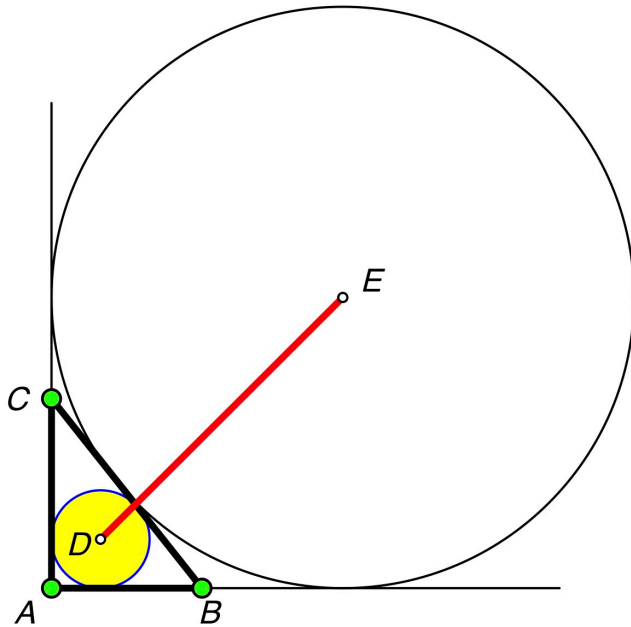
Relationships:

$\{AF \parallel CE\}$



D is incenter of $\triangle ABC$
E is circumcenter of $\triangle ABC$

Prove: AEBC cyclic

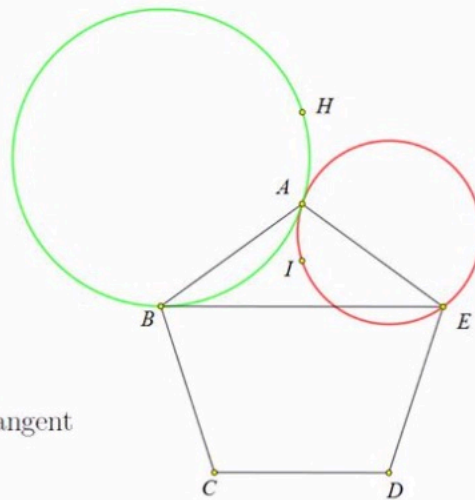


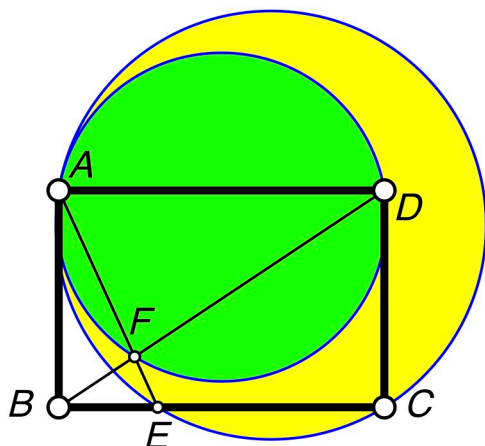
Start with right triangle ABC
 1. $D = \text{incenter}[ABC]$
 2. $E = \text{excenter}[ABC, A]$
 Conclusion: $DE/BC = \sqrt{2}$

- ★ $ABCDE$ regular pentagon
- ★ I incenter of $\triangle ABE$
- ★ H orthocenter of $\triangle ABE$

Prove that:

$\odot(BAH)$ and $\odot(AIE)$ are tangent





ABCD is a rectangle.
 E is any point on BC.
 $F = BD \cap AE$.

Prove:

$\odot ACE$ is tangent to $\odot ADF$.

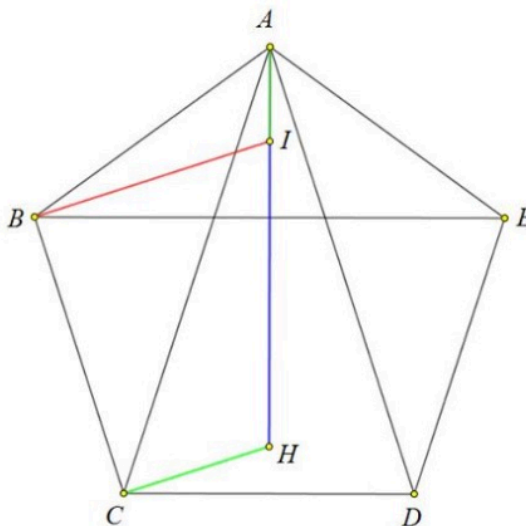
Stanley Rabinowitz, December 2019

4076

- ★ $ABCDE$ regular pentagon
- ★ I incenter of $\triangle ABE$
- ★ H orthocenter of $\triangle ACD$

Prove that:

- ◆ $BI \parallel CH$
- ◆ $CH + BI = AH$
- ◆ $CH + AI = BI$
- ◆ $\frac{1}{CH} + \frac{1}{BI} = \frac{1}{AI}$



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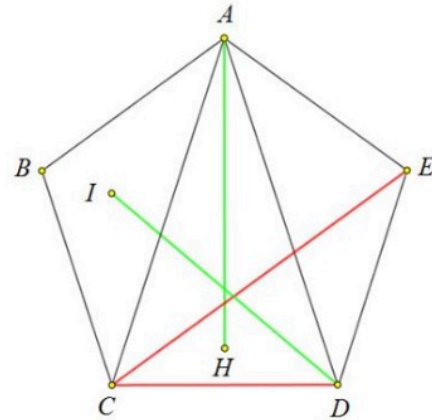


4084

★ $ABCDE$ regular pentagon

★ I incenter of $\triangle ABC$

★ H orthocenter of $\triangle ACD$

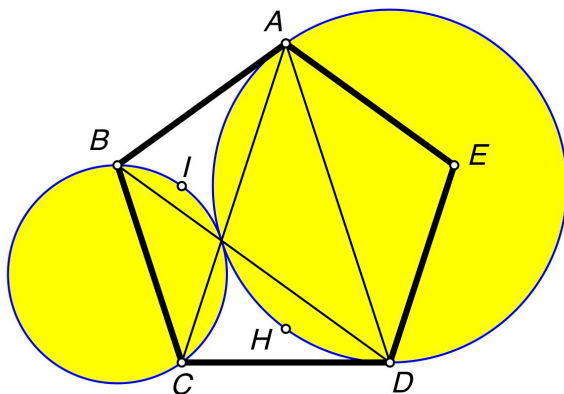


Prove that:

◆ $CD^2 + CE^2 = AH^2 + DI^2$

◆ $\frac{1}{CH} + \frac{1}{CI} = \frac{1}{BI}$

ERCOLE SUPPA 10/12/2019

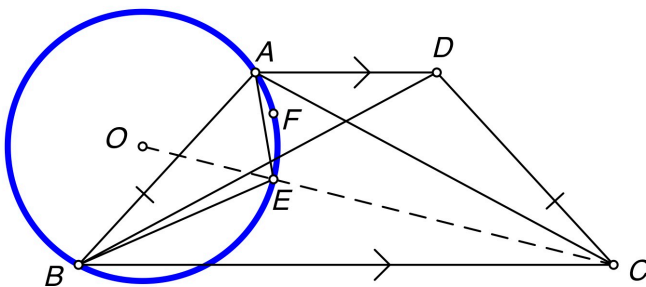


$ABCDE$ is a regular pentagon.
 I is the incenter of $\triangle ABC$.
 H is the orthocenter of $\triangle ACD$.

Prove:

- (i) $\odot BCI$ is tangent to $\odot ADH$.
- (ii) AC meets BD at the touch point.

Stanley Rabinowitz, December 2019



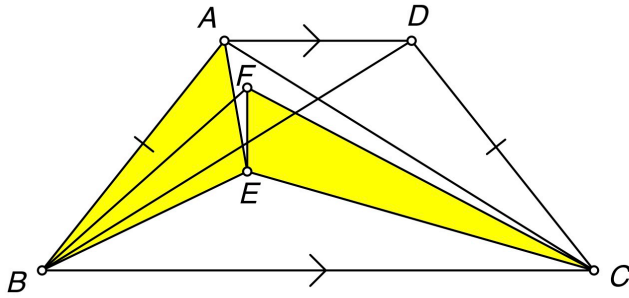
$ABCD$ is an isosceles trapezoid.

- 1. E is the incenter of $\triangle ABC$.
- 2. F is the incenter of $\triangle ABD$.

Prove:

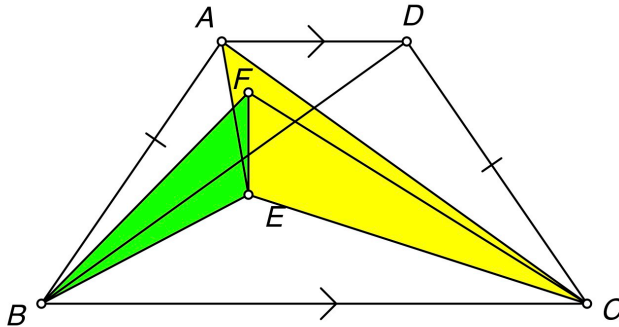
- (i) $ABEF$ is cyclic.
- (ii) CEO is a straight line.

Stanley Rabinowitz, December 2019



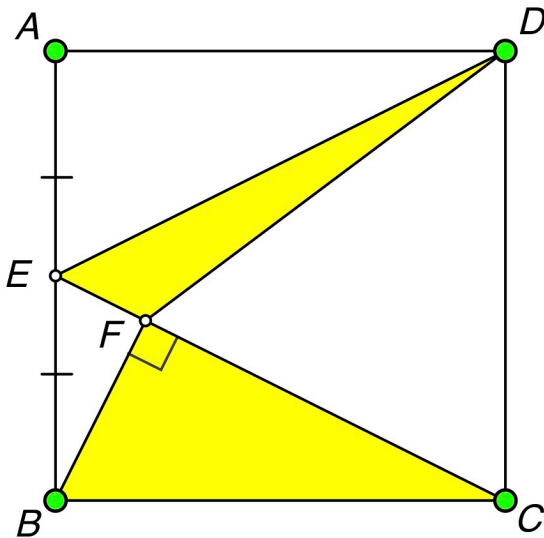
ABCD is an isosceles trapezoid.
 1. E is the incenter of $\triangle ABC$.
 2. F is the incenter of $\triangle ABD$.
 Prove: Triangles ABE and CEF have the same area.

Stanley Rabinowitz, December 2019



ABCD is an isosceles trapezoid.
 1. E is the incenter of $\triangle ABC$.
 2. F is the incenter of $\triangle ABD$.
 Prove: $\triangle ACE \sim \triangle FBE$.

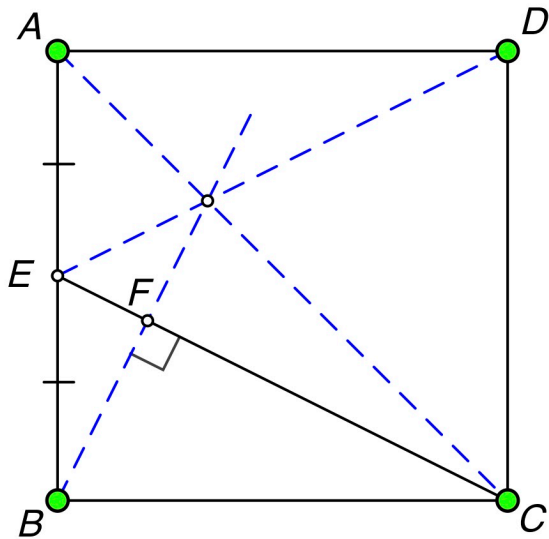
Stanley Rabinowitz, December 2019



ABCD is a square.
 $AE = BE$.
 $BF \perp CE$.

Prove: $\triangle BCF$ and $\triangle DEF$ have the same perimeter.

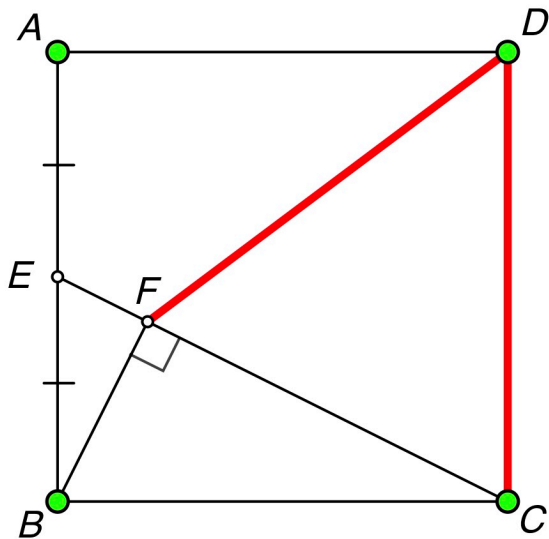
Stanley Rabinowitz, December 2019



ABCD is a square.
 AE=BE.
 BF ⊥ CE.

Prove: AC, BF, DE concur.

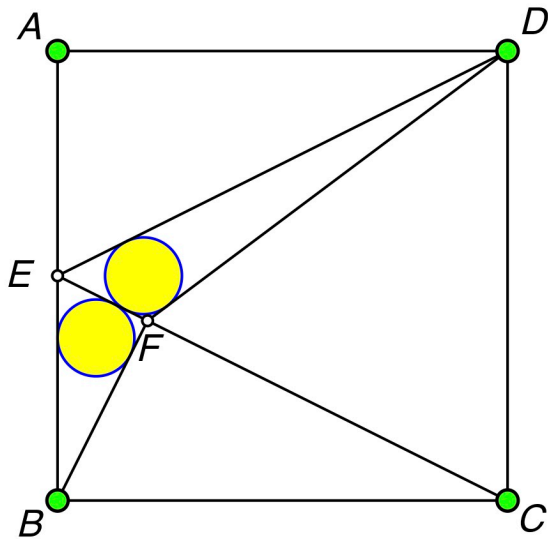
Stanley Rabinowitz, December 2019



ABCD is a square.
 AE=BE.
 BF ⊥ CE.

Prove: DF=DC.

Stanley Rabinowitz, December 2019



$ABCD$ is a square.
 $AE = BE$.
 $BF \perp CE$.

Prove: incircles of $\triangle BEF$
 and $\triangle DEF$ are congruent.

Stanley Rabinowitz, December 2019

4094

★ $ABCD$ square with center O

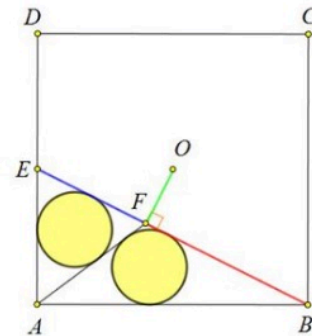
★ $AE = ED$

★ $OF \perp EB$

Prove that:

◆ $EF + FO = FB$

◆ the incircles of $\triangle AFE$ and $\triangle ABF$ are congruent



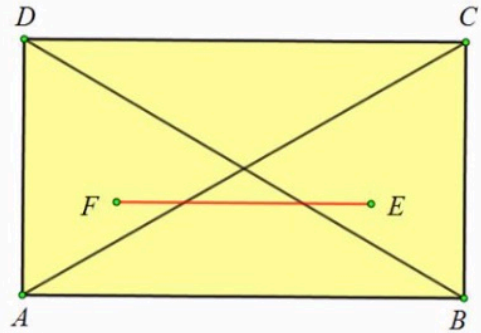
ERCOLE SUPPA 11/12/2019



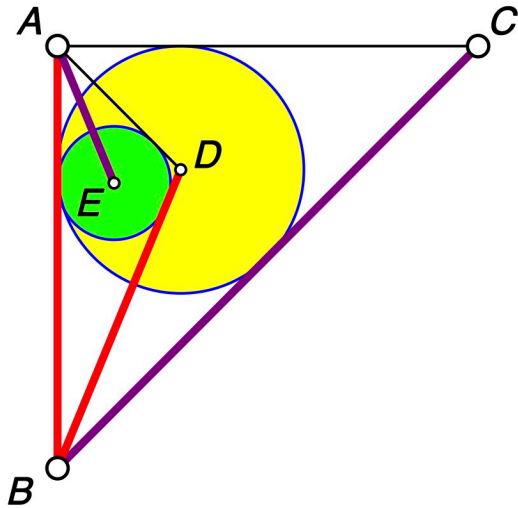
4100 - An easy problem 😊

- ★ $ABCD$ rectangle
- ★ E incenter of $\triangle ABC$
- ★ F incenter of $\triangle ABD$

Prove that: $AD + EF = AC$



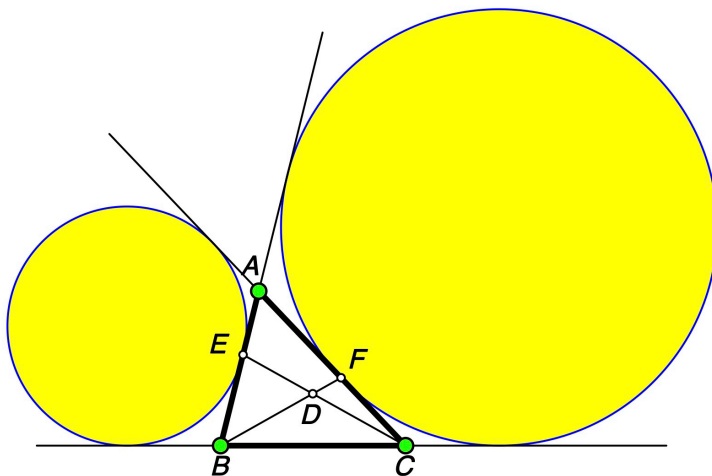
ERCOLE SUPPA 13/12/2019



$BA=AC, BA \perp AC$
 D is the incenter of $\triangle BAC$.
 E is the incenter of $\triangle BAD$.

Prove: $AB+BD=AE+BC$

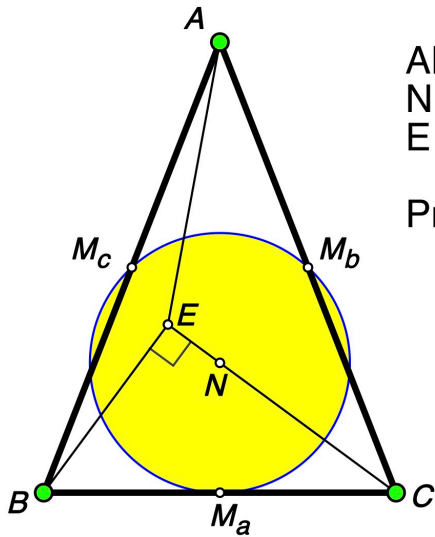
Stanley Rabinowitz, December 2019



ABC is an arbitrary triangle.
 E and F are touch points.

Prove: $AC \cdot DF = AE \cdot BD$

Stanley Rabinowitz, December 2019

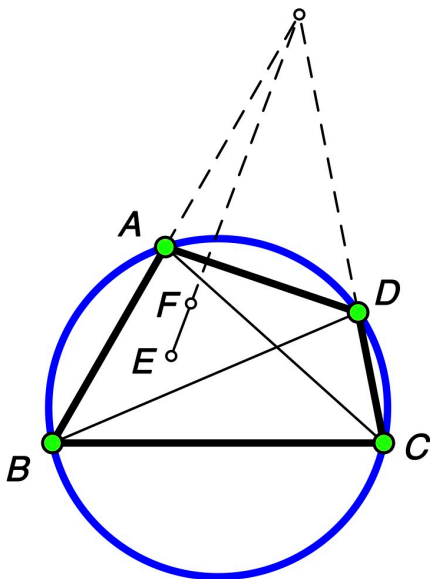


$AB=AC$

N is the center of the 9-point circle of $\triangle ABC$.
 E is the foot of the perpendicular from B to CN .

Prove: $AB \cdot BE = AE \cdot BC$

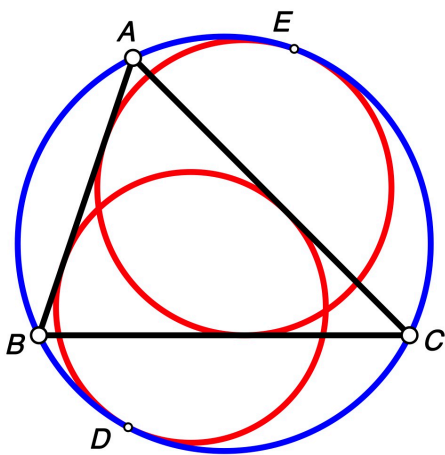
Stanley Rabinowitz, December 2019



$ABCD$ is a cyclic quadrilateral.
 E is the symmedian point of $\triangle ABC$.
 F is the symmedian point of $\triangle ABD$.

Prove: BA, CD, EF concur

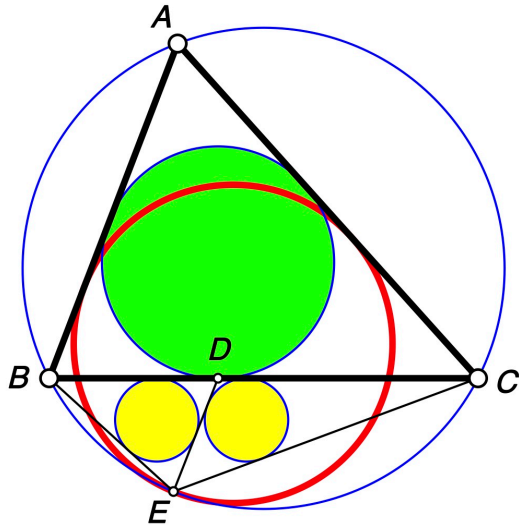
Stanley Rabinowitz, December 2019



The blue circle is the circumcircle of $\triangle ABC$.
 Points D and E are touch points.

Prove: $AD \cdot CE = BE \cdot CD$

Stanley Rabinowitz, January 1, 2020



D is the point where the incircle of $\triangle ABC$ touches BC.
 E is the point where the red circle touches the circumcircle of $\triangle ABC$.

Prove: The incircles of $\triangle BDE$ and $\triangle CDE$ are congruent.

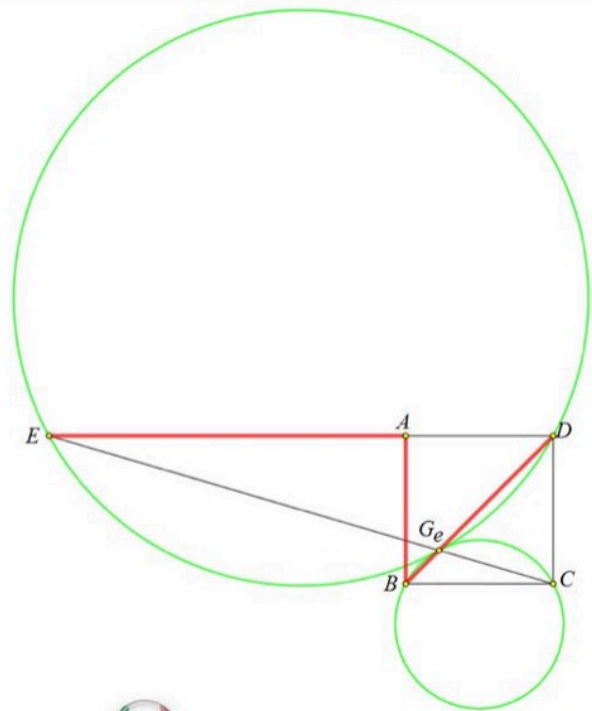
Stanley Rabinowitz, January 2020

4271

- * $ABCD$ square
- * G_e Gergonne point of $\triangle ABC$
- * $E = AD \cap CG_e$

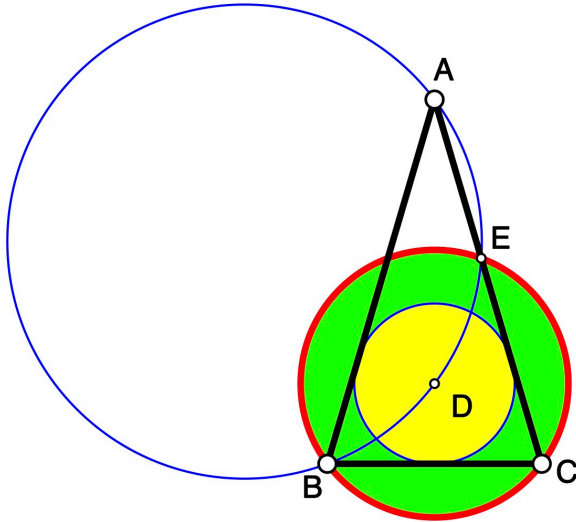
Prove that:

- ♦ $AB + BD = EA$
- ♦ $\odot(DAG_e)$ and $\odot(BCG_e)$ are tangent



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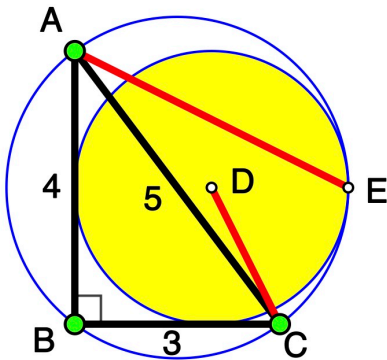




$AB=AC$
 D is incenter of $\triangle ABC$.
 $\odot ABD$ meets AC at E .

Prove: $\odot BCE$ and the incircle
 are concentric.

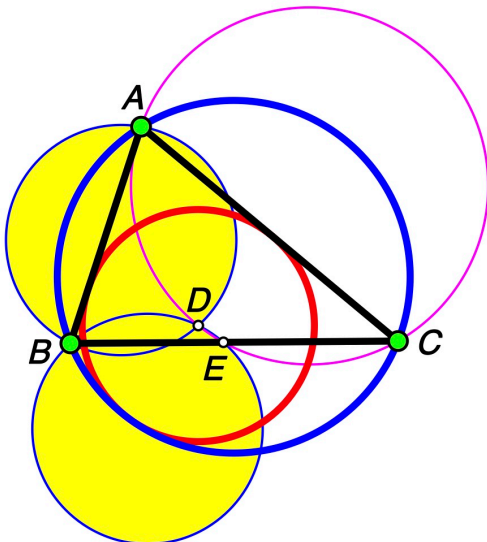
Stanley Rabinowitz, January 2020



ABC is a 3-4-5 right triangle.
 D is the center of the circle that touches
 AB , BC , and the circumcircle of $\triangle ABC$.
 E is the touch point of the two circles.

Prove: $AE=2 \cdot CD$

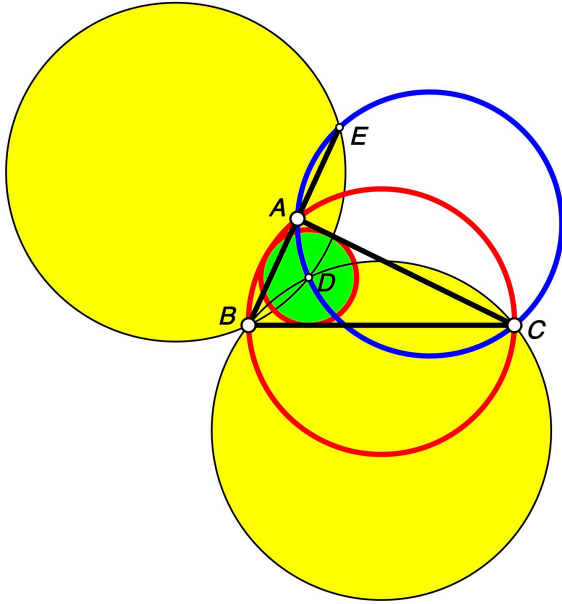
Stanley Rabinowitz, January 2020



D is the center of the red circle
 tangent to AB , AC and the blue
 circumcircle of $\triangle ABC$.
 $\odot ADC$ meets BC at E .

Prove: $\odot ABD \cong \odot BDE$
 (i.e. yellow circles are congruent)

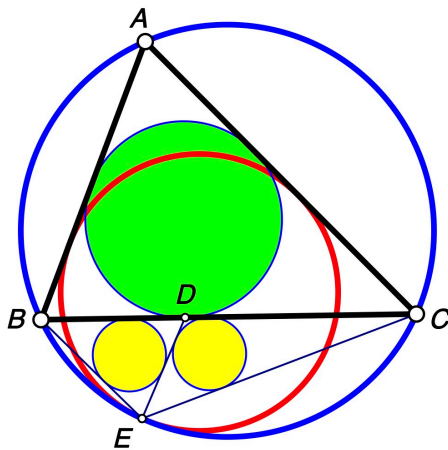
Stanley Rabinowitz, January 2020



D is the center of the green circle that touches BC, AC, and $\odot ABC$.
BA meets $\odot ADC$ at E.

Prove that $\odot BCD$ and $\odot BDE$ are congruent.

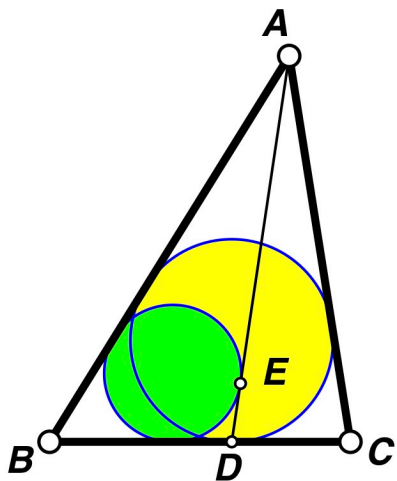
Stanley Rabinowitz, January 2020



The green incircle of $\triangle ABC$ touches BC at D.
The red circle touches the blue circle at E.

Prove that the yellow incircles are congruent.

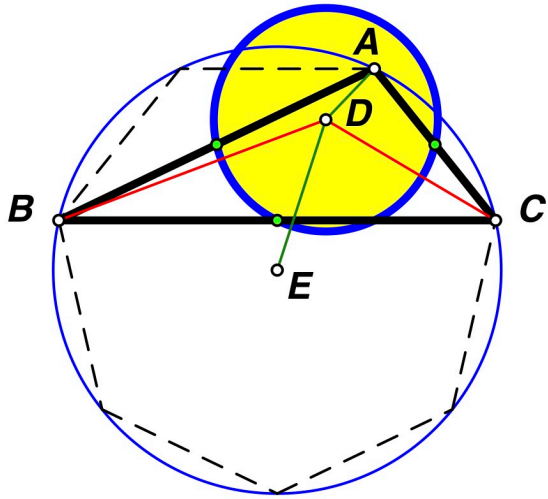
Stanley Rabinowitz, January 2020



D and E are touch points.

Prove: $AC+DE=AE+CD$

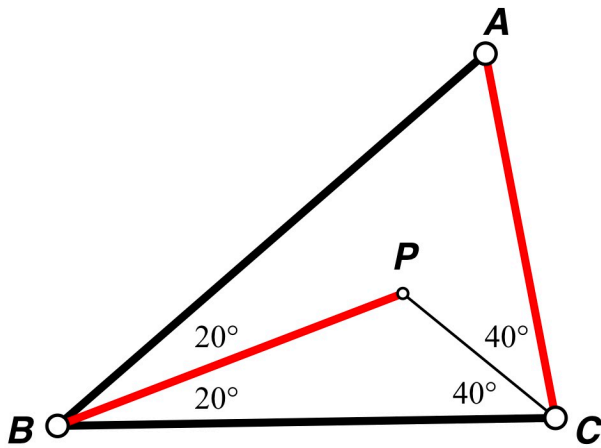
Stanley Rabinowitz, January 2020



ABC: heptagonal triangle
D: center of 9-point circle
E: circumcenter

Prove: $AD+BD=CD+DE$

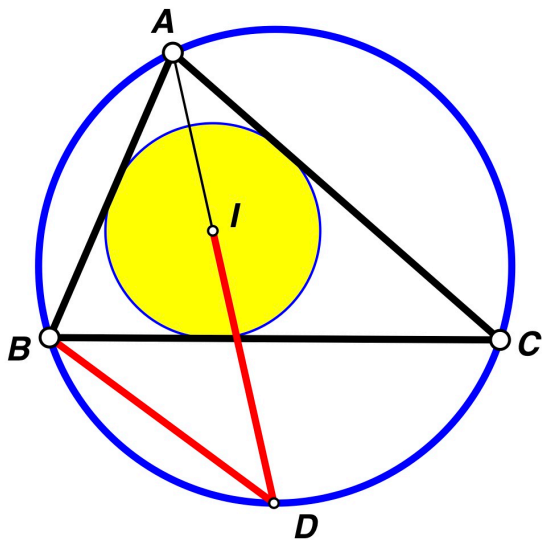
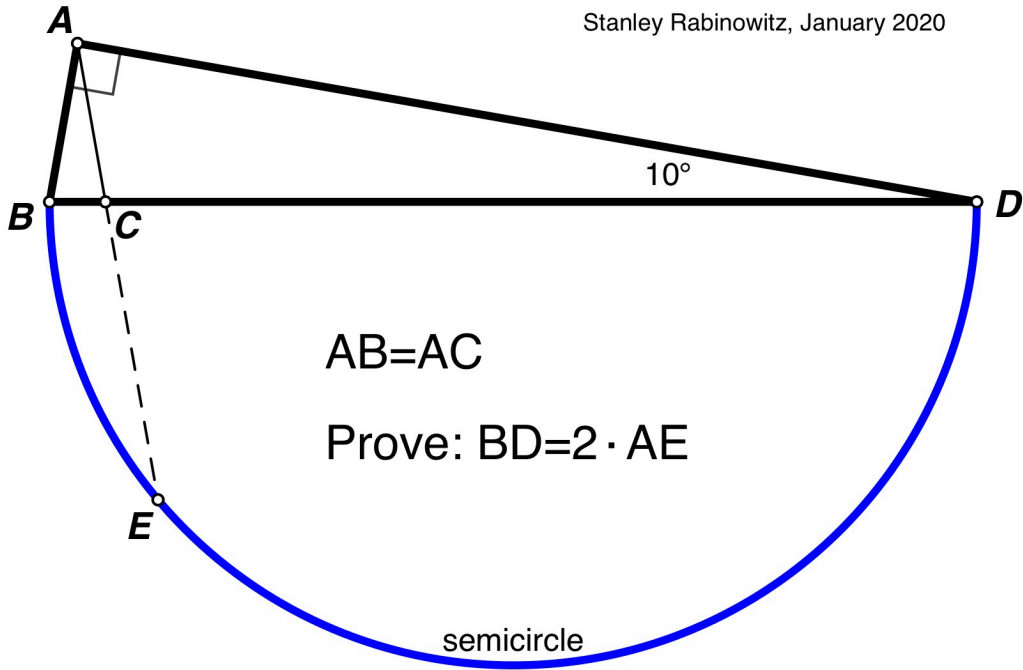
Stanley Rabinowitz, January 2020



Prove: $AC=BP$

Stanley Rabinowitz, January 2020

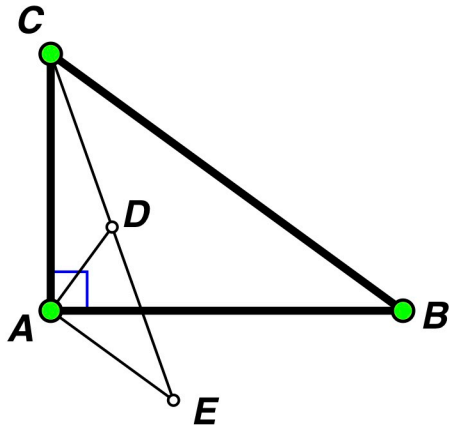
Stanley Rabinowitz, January 2020



**Let I be the incenter of $\triangle ABC$.
 AI meets $\odot ABC$ at D .**

Prove: $DB=DI$.

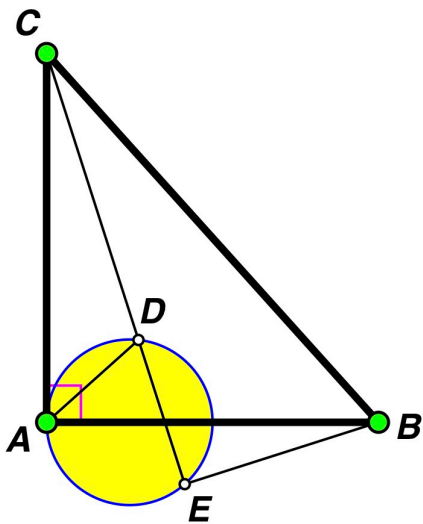
Stanley Rabinowitz, January 2020



Let D be the symmedian point of right triangle CAB .
 $CD = DE$.

Prove: $DA \perp AE$

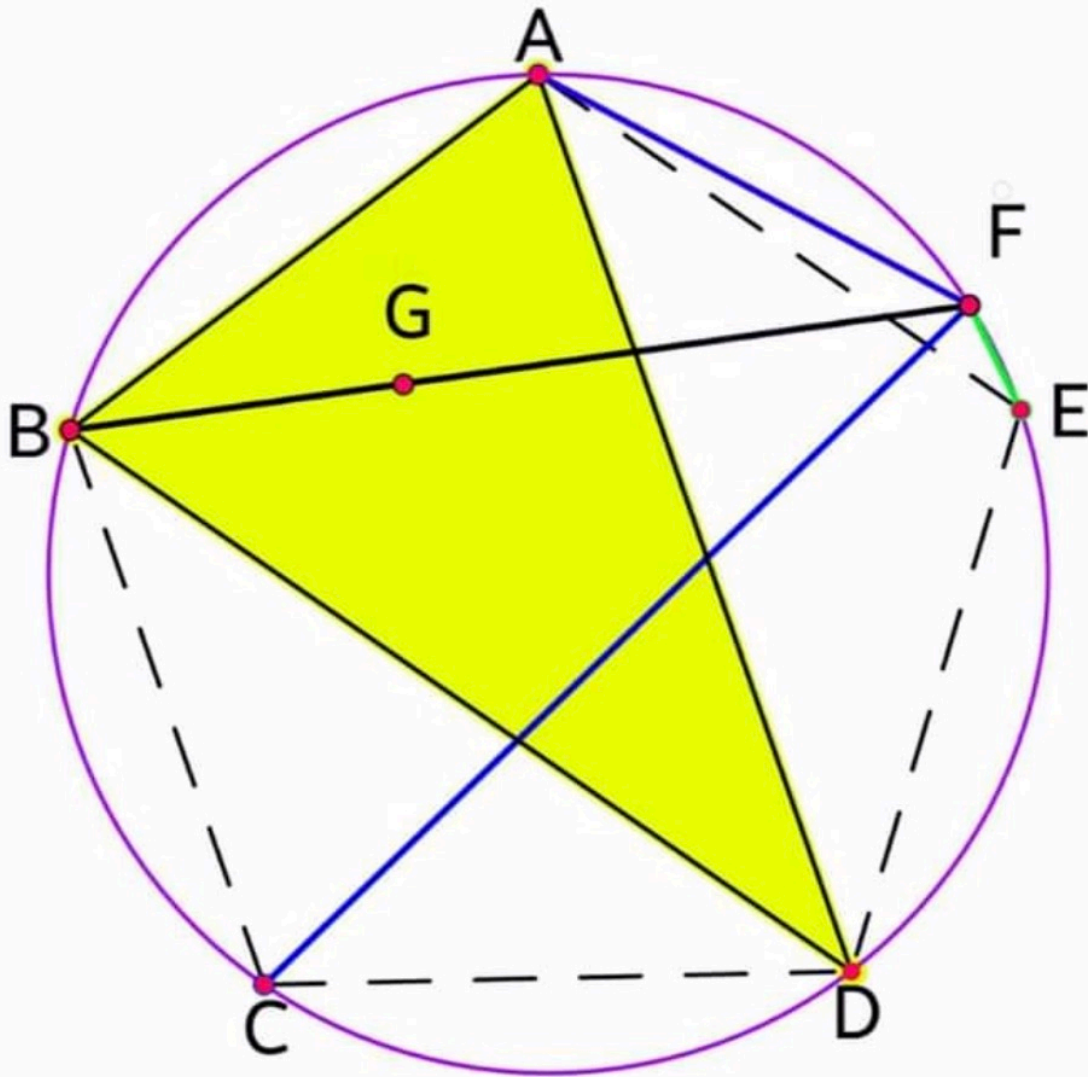
Stanley Rabinowitz, January 2020



Let D be the symmedian point of right triangle CAB .
 The circle tangent to CA at A passing through D meets CD at E .

Prove: $BE \perp CE$

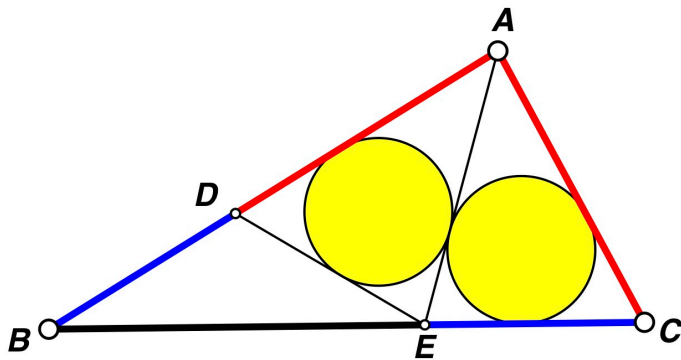
Stanley Rabinowitz, January 2020



- $ABCDE$ is a regular pentagon
- G is Gergonne point of $\triangle ABD$
- $F = \odot(ABCDE) \cap BG$

Prove :

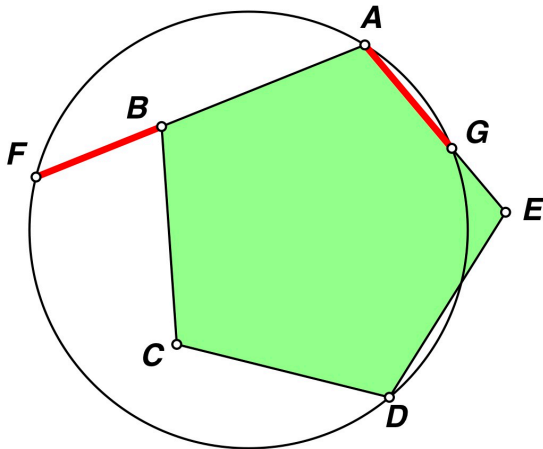
- $CF = 2AF$



$\angle ACB = 2\angle ABC$
 $AD = AC$
 $CE = BD$

Prove: Yellow circles are congruent.

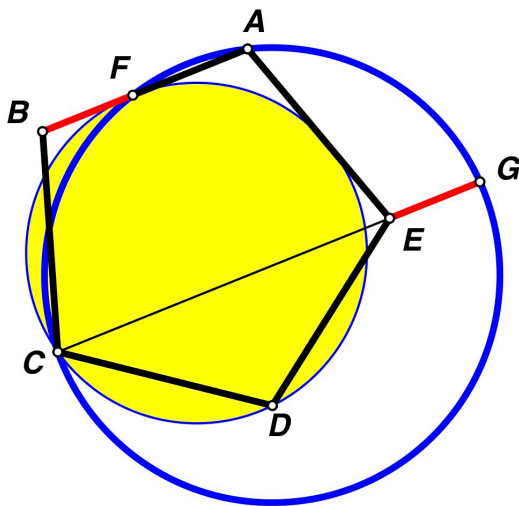
Stanley Rabinowitz, February 2020



$ABCDE$ is a regular pentagon.
 $AF = AC$
 $\odot AFD$ meets AE at G .

Prove: $BF = AG$.

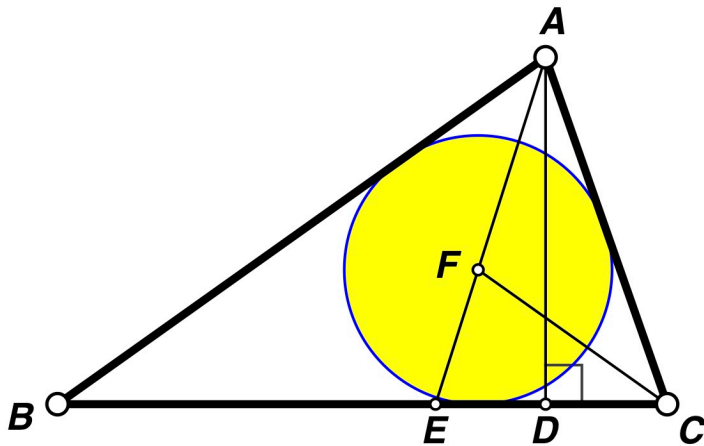
Stanley Rabinowitz, February 2020



$ABCDE$ is a regular pentagon.
 $\odot CDF$ is tangent to AB at F .
 CE meets $\odot ACF$ at G .

Prove: $BF = EG$.

Stanley Rabinowitz, February 2020

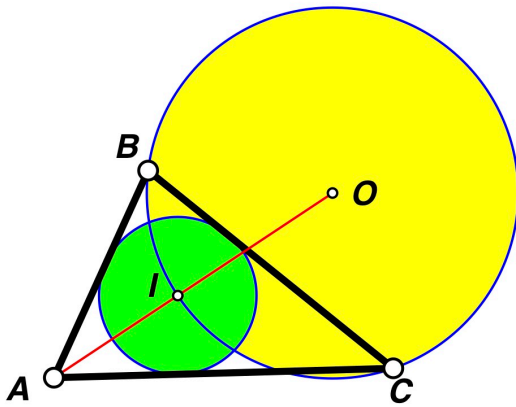


In $\triangle ABC$, $\angle C = 2\angle B$.
F is the incenter.
 $AD \perp BC$.

Prove:

- (i) $AC + CF = AB$**
- (ii) $AC + CD = BD$**
- (iii) $CE = CF$**

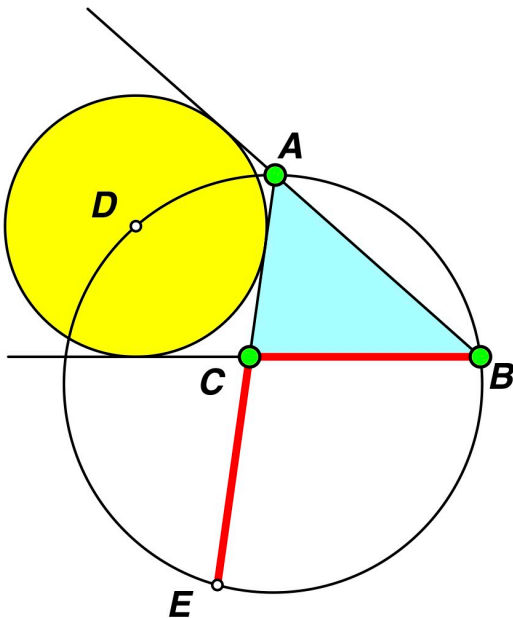
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I is the incenter of $\triangle ABC$.
O is the circumcenter of $\triangle BIC$.

Prove that A, I, O colline.

Stanley Rabinowitz, February 2020



D is the B-excenter of $\triangle ABC$.
AC meets $\odot DAB$ at **E**.

Prove $BC = CE$.

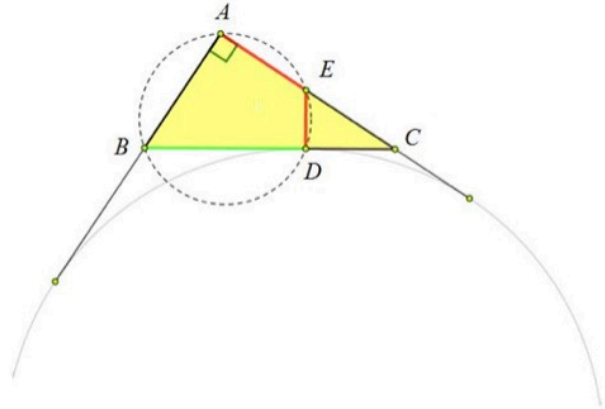
Stanley Rabinowitz, February 2020

★ $\triangle ABC$ with $\angle BAC = 90^\circ$

★ $D = (ABC, A)$ extouch

★ $\odot(ABD) \cap AC = \{A, E\}$

Prove that: $AE + ED = BD$



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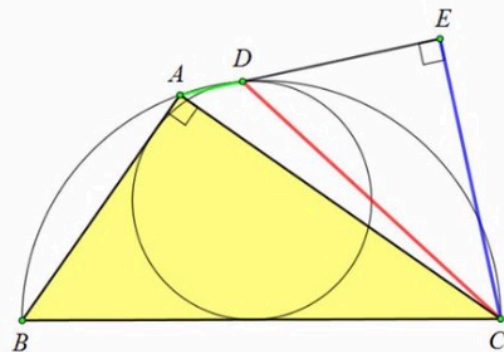
4577

★ $\triangle ABC$ with $\angle BAC = 90^\circ$

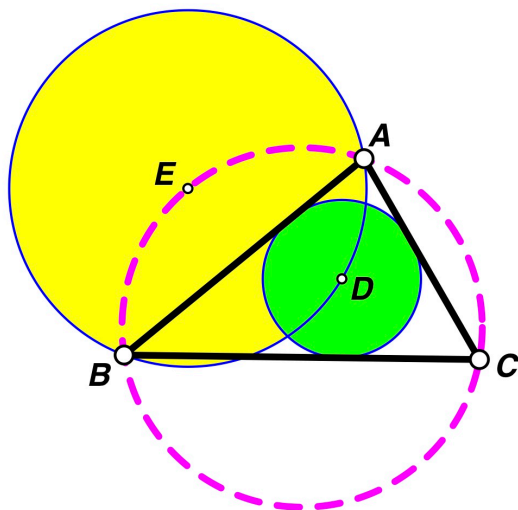
★ $D = (ABC, B)$ mixtouch

★ E foot of C on AD

Prove that: $AD + CE = CD$



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**D is the incenter of $\triangle ABC$.
E is the circumcenter of $\triangle ABD$.**

Prove AEBC concyclic..

Stanley Rabinowitz, February 2020

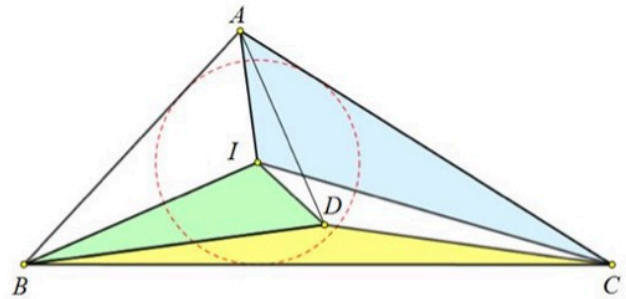
4589

★ I incenter of $\triangle ABC$

★ D orthocenter of $\triangle ABI$

Prove that:

$$[BDI] + [BCD] = [AIC]$$



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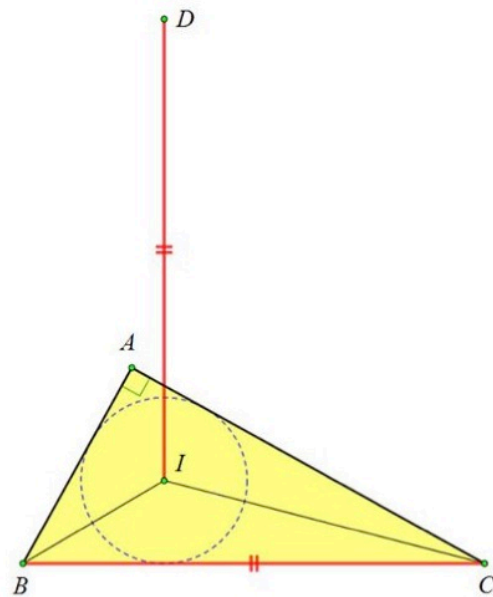
4590 - Inspired by Stanley Rabinowitz problem # 4578

★ $\triangle ABC$ right triangle with $\angle A = 90^\circ$

★ I incenter of $\triangle ABC$

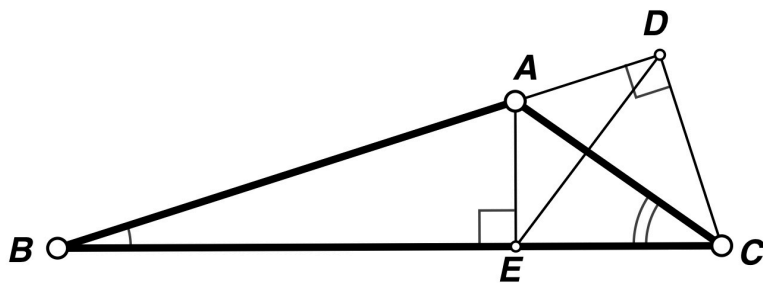
★ D orthocenter of $\triangle BCI$

Prove that: $DI = BC$



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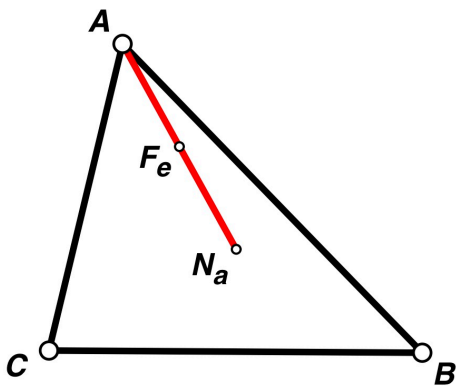




$\angle ACB = 2\angle ABC$
 $AE \perp BC, CD \perp BA$

Prove: $AB = 2DE$

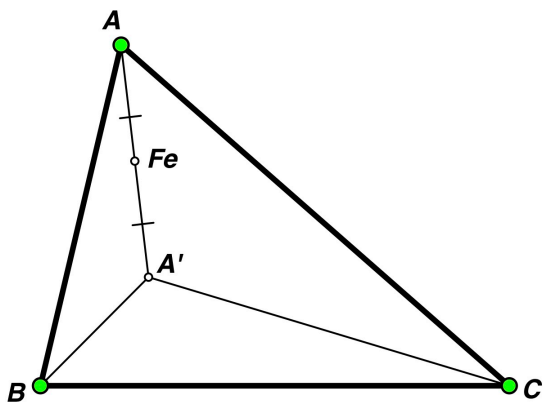
Stanley Rabinowitz, February 2020



$AB + AC = 2 \cdot BC$
 N_a is the Nagel point of $\triangle ABC$.
 F_e is the Feuerbach point of $\triangle ABC$.

Prove: F_e is the midpoint of AN_a

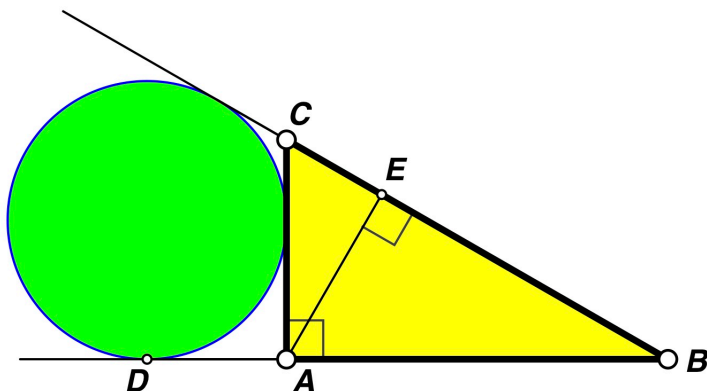
Stanley Rabinowitz, February 2020



Let F_e be the Feuerbach point of $\triangle ABC$.
 Reflect A about F_e to get A' .

Prove: $\angle BAC + \angle BA'C = 180^\circ$

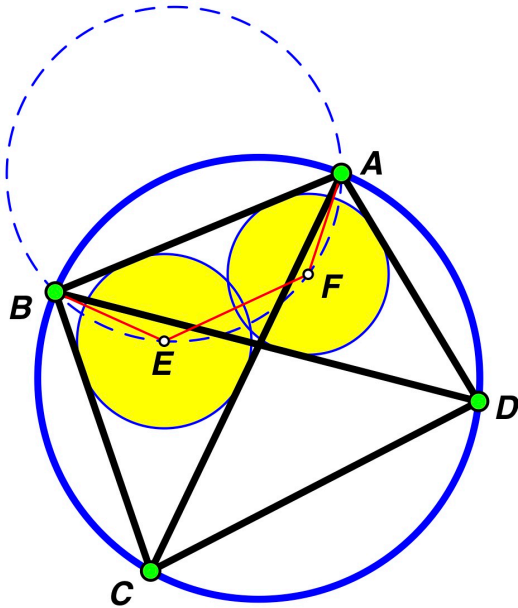
Stanley Rabinowitz, March 2020



Prove:

$$\frac{1}{AB} + \frac{1}{AD} = \frac{1}{AC} + \frac{1}{AE}$$

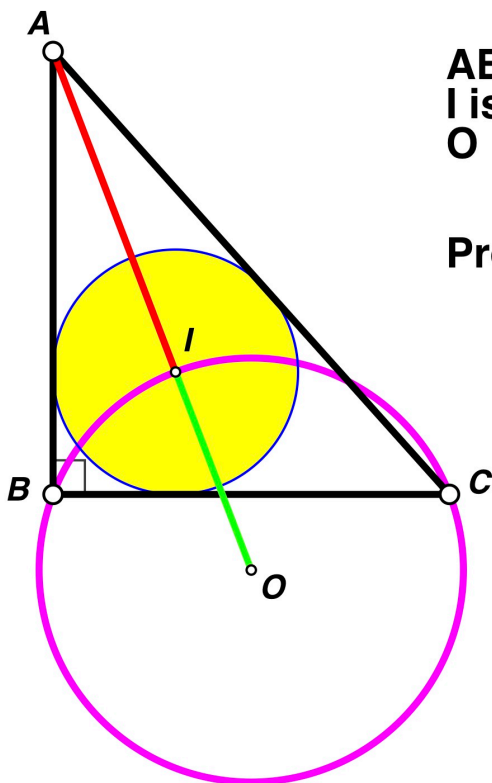
Stanley Rabinowitz, March 2020



**ABCD is a cyclic quadrilateral.
E is the incenter of $\triangle ABC$.
F is the incenter of $\triangle ABD$.**

Prove: A, B, E, F concyclic.

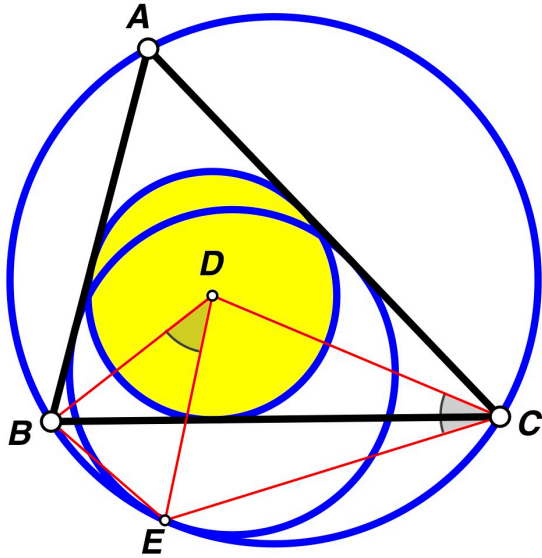
Stanley Rabinowitz, March 2020



**ABC is a right triangle with $3BC=2AC$.
I is the incenter of $\triangle ABC$.
O is the circumcenter of $\triangle BIC$.**

Prove: $\frac{AI}{IO} = \phi$, the golden ratio

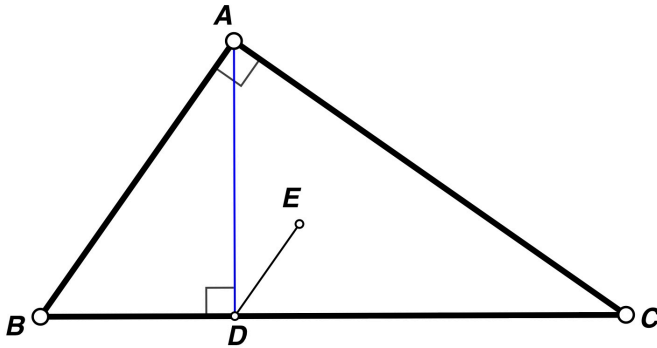
Stanley Rabinowitz, March 2020



D is the incenter of $\triangle ABC$.
E is the A-mixtilinear touch point of $\triangle ABC$.

Prove: $\angle BDE = \angle DCE$

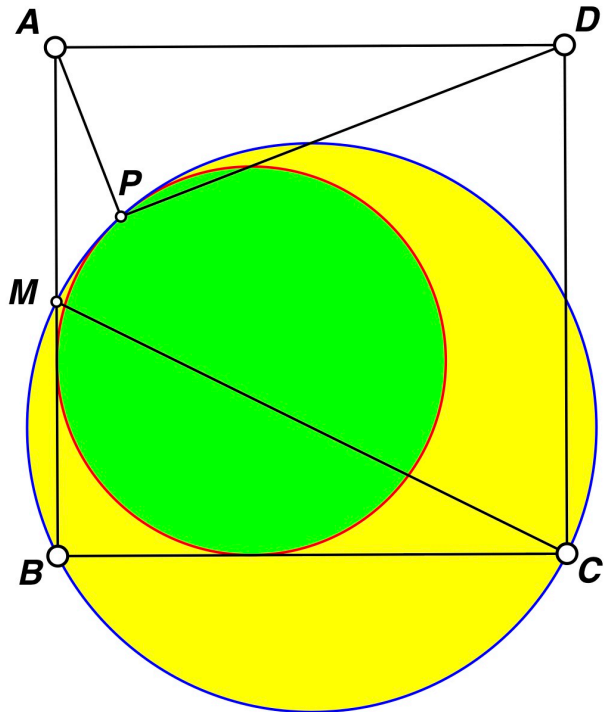
Stanley Rabinowitz, March 2020



$AB \perp AC$
 $AD \perp BC$
E is symmedian point of $\triangle ADC$.

Prove: $ED \parallel AB$

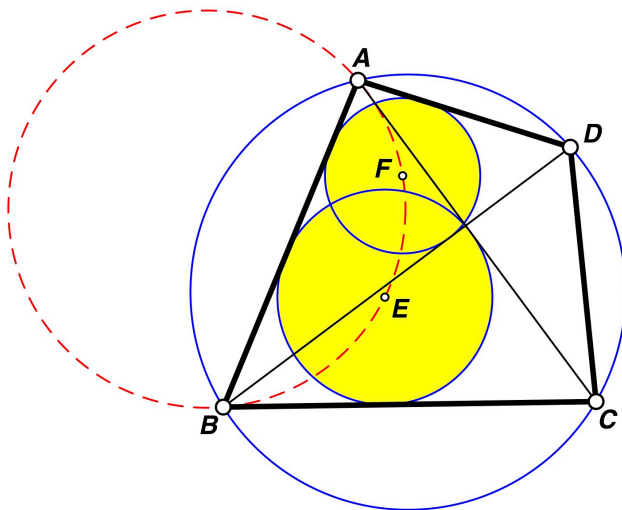
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**ABCD is a square.
 $AM=MB$.
 P is the B-mixtilinear
 touch point of $\triangle MBC$.**

Prove: $AP \perp DP$

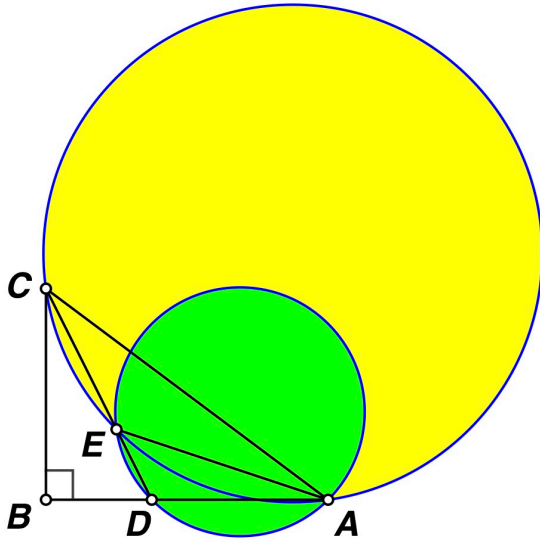
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**ABCD is a cyclic quadrilateral.
 E is the incenter of $\triangle ABC$.
 F is the incenter of $\triangle ABD$.**

Prove: ABEF is cyclic

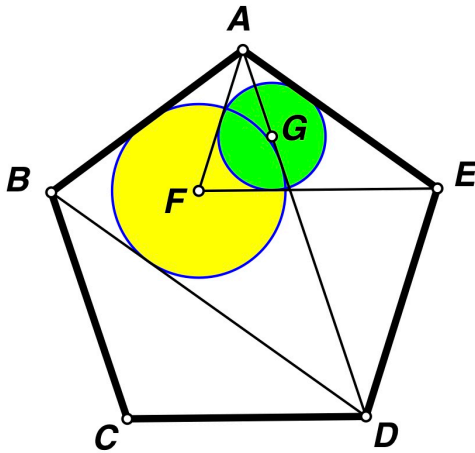
Stanley Rabinowitz, April 2020



**$BC=3, AB=4, AC=5$
 CD bisects $\angle ACB$
 AE bisects $\angle BAC$**

**Prove: radius of $\odot ACE$ is
 twice the radius of $\odot ADE$.**

Stanley Rabinowitz, April 2020



**$ABCDE$ is a regular pentagon.
 F is the incenter of $\triangle ABD$.
 G is the incenter of $\triangle AEF$.**

Prove:

$$\frac{\text{radius of } \odot F}{\text{radius of } \odot G} = \phi$$

Stanley Rabinowitz, April 2020